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Problem Solving in Three-Dimensional Geometry: How Are Pre-Service Mathematics Teachers Mathematical Communication Characteristics?

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Abstract: In solving mathematics problems, pre-service mathematics teachers differ in their communication styles. However, no recent research has focused on classifying pre-service mathematics teachers' mathematical communication styles for solving non-routine problems, particularly those in three-dimensional geometry. The present research is a descriptive qualitative which aim to analyze pre-service mathematics teachers' mathematical communication skills and classifying them into three categories. The subjects consisted of pre-service mathematics teachers willing to be volunteers, employed mathematical communication in problem solving, and could implement think-aloud approaches in their mathematical problem solving. Data collection methods include non-routine tests, observations, and in-depth interviews. Data validation techniques included source triangulation. The results showed that the pre-service mathematics teachers engaged in three categories of mathematical communication: notation, visual, and argumentation. Pre-service mathematics teachers in the notation category tended to represent mathematical problems in the form of notations, such as mathematics symbols. In the second visual category, the students visualised the information into a picture or a diagram. The third category was argumentation, which consisted of pre-service mathematics teachers who tended to answer questions by presenting logical and systematic arguments.

Keywords: Mathematics communication, non routine problems, Pre-service mathematics teachers, Three-dimensional geometry.

1. Introduction

There are many forms of communication in everyday life, including communication in the classroom. Communication may occur in interactions between teachers and pre-service mathematics teachers or among pre-service mathematics teachers. Communication is an essential aspect of mathematics. According to Smieskova (2017), mathematical communication skills are valuable for developing pre-service mathematics teachers creativity and motivation. It is fundamental to success in mathematics that both pre-service mathematics teachers and teachers possess these skills. By communicating, pre-service mathematics teachers may communicate their ideas and reflect on their problem-solving activities, which strengthens their understanding of mathematics.

According to the National Council of Teachers of Mathematics (NCTM) (2000), the skills that pre-service mathematics teachers need to learn through mathematics are (1) problem solving, (2) reasoning and proof, (3) connection, (4) communication, and (5) representation. Communication skills are important in mathematics learning and comprise a standard process in mathematics. Mathematical communication skills refer to pre-service mathematics teachers ability to (1) organise and connect their mathematical thinking through communication; (2) communicate their mathematical thinking logically and clearly to friends, teachers, and others; (3) analyse and assess mathematical thinking and strategies used by others; and (4) use mathematical language to express mathematical ideas correctly (NCTM, 2000). Such skills are an important requirement for improving mathematics learning outcomes among pre-service mathematics teachers. Therefore, mathematical skills (including mathematical communication) still need to be developed through the integration of fun learning so that problem solvers can communicate innovative ideas in mathematical problem-solving activities (Sutama et al., 2021; 2022). Mathematical skills mentioned among reflective thinking and communication (Kholid, et al., 2022).

Mathematical communication is a way for pre-service mathematics teachers to express mathematical ideas orally, in writing, pictures, or diagrams, through objects, in algebraic form, or using mathematical symbols. Through mathematical communication, pre-service mathematics teachers may convey ideas and clarify their understanding and knowledge during the mathematics learning process (Disasmitowati & Utami, 2017; NCTM, 2000). Mathematical communication is one of the skills all pre-service mathematics teachers need because it enables them to understand mathematics through thinking, discussing, and making decisions (Viseu & Oliveira, 2012). According to Guerreiro and Serrazina (2010), mathematical communication is a tool for transmitting mathematical knowledge. Communication makes mathematical thinking observable, and as a result, it facilitates the development of mathematical problem solving. For effective mathematical communication, one must interpret and generate mathematical ideas verbally or non-verbally in careful, analytical, and critical ways to evaluate and process them into in-depth understanding (Saputra et al., 2022; Kholid et al., 2020). Mathematical communication skills help pre-service mathematics teachers develop a conceptual understanding of a particular mathematics problem (Kholid et al., 2021). Moreover, it provides opportunities for pre-service mathematics teachers to clarify their understanding and consolidate mathematical ideas (Hirschfeld-Cotton, 2008).

According to National Education Ministry Law (2006) number 22, concerning Content Standards for Mathematics Learning, one of the goals of learning mathematics is the ability to communicate ideas about the mathematical objects being studied. In line with this goal, mathematical communication is essential in formulating pre-service mathematics teachers' ideas. Therefore, pre-service mathematics teachers need to be trained in mathematical communication skills.

According to Abidin (2019) characteristics of mathematical communication, based on its characteristics, it is divided into two, namely oral and written. There are three types of oral mathematical communication, namely lecture, instructional, and discussion. Meanwhile, in written mathematical communication there are also three types, namely narrative, procedure and dialogue.

In communicating mathematical ideas, pre-service mathematics teachers may experience obstacles. As inadequate mathematical knowledge is a significant barrier to developing mathematical communication skills, this may indicate that pre-service mathematics teachers lack those skills. In this case, teachers need to encourage pre-service mathematics teachers to apply mathematization so that pre-service mathematics teachers can communicate and solve math problems (Kholid, et al., 2022). The next most important aspect of mathematical communication is mathematical vocabulary. With limited mathematical vocabulary, the mathematical communication process cannot progress smoothly, which becomes a barrier to effective communication. In addition, the level of pre-service mathematics teachers' self-confidence is an essential factor in mathematical communication. For instance, pre-service mathematics teachers not brave enough to express their ideas tend not to actively. Interest in learning also has a significant influence on mathematical communication skills. Pre-service mathematics teachers with a high interest in learning will be better at communicating math problems than those with a low interest in learning (Kholid et al., 2019). Masduki et al. (2020) state that metacognitive abilities affect pre-service mathematics teachers mathematical communication skills. Pre-service mathematics teachers can control problem-solving activities by optimizing metacognitive skills so that the problem-solving process runs optimally.

Nasrullah & Baharman (2017) stated that indicators or activities included in mathematical communication include: (1) Expressing a situation, image, diagram or real object into language, symbols, ideas or mathematical models. (2) Explain mathematical ideas, situations and relationships orally or in writing. (3) Listen, discuss, and write about mathematics. (4) Understand written mathematical representations. (5) Rephrase a mathematical description or paragraph in your own language. Meanwhile, NCTM, 2000 states that mathematics communication standards are mathematics learning that focuses on students' abilities to: (1) Organize and strengthen mathematical thinking through communication. (2) communicate mathematical thinking consistently (in a logical sequence) and explain to friends, teachers and other people. (3) Analyze and evaluate other people's mathematical thinking and strategies. (4) Using mathematical language to express mathematical concepts correctly.

1.1 Research Position

Researchers reviewed several studies related to mathematical communication, and they grouped them into three main categories: (1) research on the correlation between mathematical learning and communication models, (2) research on the correlation between learning devices and mathematical communication, and (3) research analysing communication skills in mathematics.

The first group included studies on the correlation between learning models and mathematical communication. The results concluded that (1) cooperative learning type team-assisted individualisation (TAI) affects mathematical communication skills (Tinungki, 2015); (2) the implementation of mathematics learning with the Realistic Mathematics Education (RME) approach may improve mathematical communication skills (Trisnawati et al., 2018); (3) contextual learning can improve communication between pre-service mathematics teachers mathematical abilities (Nartani et al., 2015); and (4) mathematics learning using the Brain Based Learning (BBL) approach with autographs contributes to developing mathematical communication skills (Triana et al., 2019).

The second group consisted of research on the correlation between learning tools and mathematical communication, which reported the following findings: (1) GeoGebra-assisted reciprocal peer tutoring strategies significantly affect pre-service mathematics teachers mathematical communication skills (Lestari et al., 2019); (2) there is an observable improvement in pre-service mathematics teachers mathematical and social skills by using problem-based learning tools in the Acehnese cultural context (Aufa et al., 2016); (3) the application of generative learning with teaching aids improves mathematical communication skills (Wardono et al., 2020); and (4) there is a significant difference in the mathematical communication skills of pre-service mathematics teachers assisted by GeoGebra (Kusumah et al., 2020).

The third stream of research analysed the mathematical communication skills of pre-service mathematics teachers. The results indicated the following: (1) pre-service mathematics teachers still experience many difficulties in communicating and presenting their mathematical content (Uyen et al., 2021); (2) effective communication occurs in the classroom if it is a critical component of pre-service mathematics teachers learning from the start of instruction (Olteanu & Olteanu, 2013); (3) pre-service mathematics teachers have different difficulties in communicating problems (Baran & Kabael, 2021); (4) reciprocal teaching strategies are most effective at improving pre-service mathematics teachers mathematical communication (Qohar & Sumarmo, 2013); (5) generally, pre-service mathematics teachers mathematical communication skills still need to be developed (Rohid et al., 2019), and (6) pre-service mathematics teachers mathematical communication skills can be categorised into several levels with different attributes (Ikhsan et al., 2020).

After reviewing the research on mathematical communication and grouping it into these three categories, the researchers found several areas that need investigation, including the classification of mathematical communication, factors that influence mathematical communication, and the development of IT-based learning media to improve pre-service mathematics teachers' mathematical communication. This year, our research focused on the classification of mathematical communication. Figure 1 illustrates the past research we have conducted and future research that we have planned.

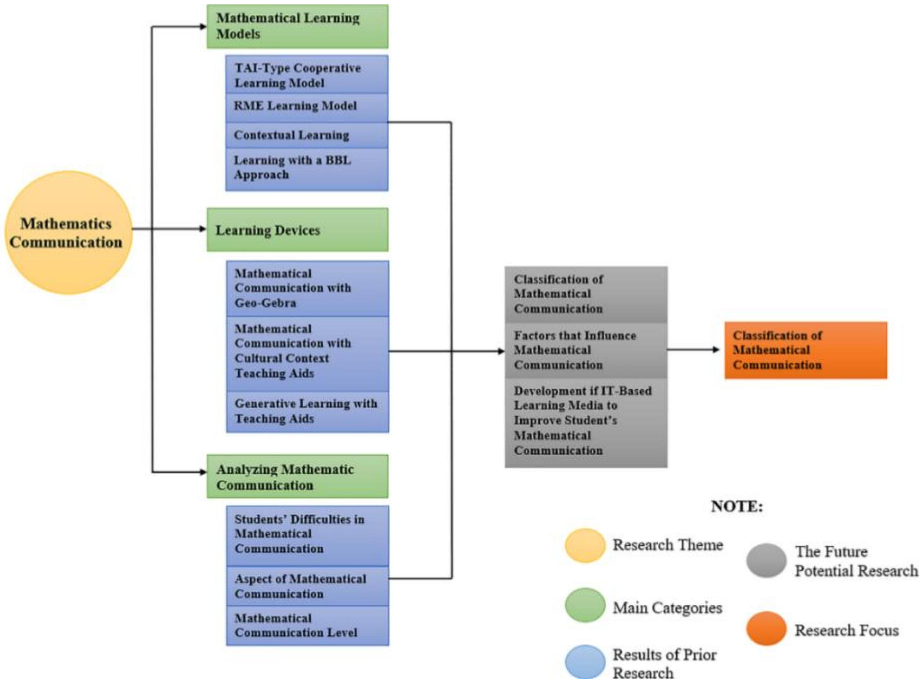


Fig. 1 Research Position

1.2 Research Roadmap

In 2015, the researchers conducted a study on implementing the problem-based learning (PBL) learning model based on assessment for learning (AfL) to improve mathematical communication. Next, in 2020, researchers conducted a jigsaw learning model experiment with an AfL-based assessment of pre-service mathematics teachers' mathematical communication skills. In 2022, the researchers studied the defragmentation of pre-service mathematics teachers' mathematical communication in solving higher-order thinking skills (HOTS) problems. The findings will be employed as a guideline in 2023 to conduct further research on pre-service mathematics teachers' mathematical communication classification. The research plan for 2024 is to focus on factors that affect pre-service mathematics teachers' mathematical communication skills, and in 2025, the objective will be to develop IT-based learning media to improve pre-service mathematics teachers mathematical communication.

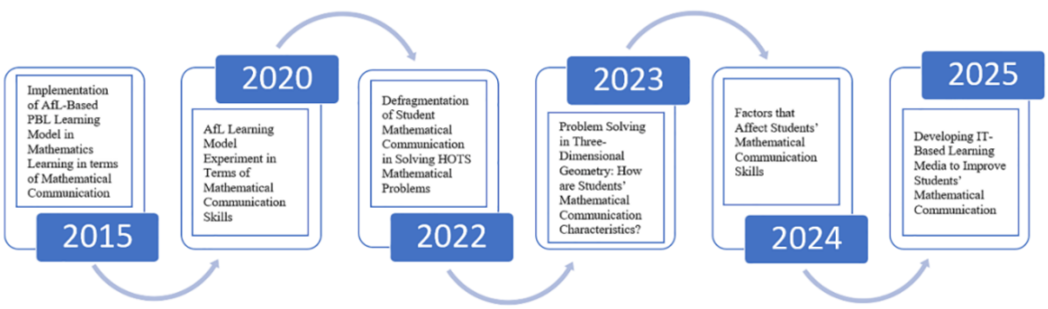


Fig. 2 Research Roadmap

1.3 Research Questions and Urgency

The question in this research is how are categorization of pre-service mathematics teachers communication for mathematical non-routine problem solving. The study aimed to categorize pre-service mathematics teachers' mathematical communication skills as they are used in solving mathematical problems, particularly non-routine ones. The results of this study may be used as a guide to describing pre-service mathematics teachers' mathematical communication skills in mathematical problem solving. In addition, this study can be considered a theoretical foundation for further research to improve pre-service mathematics teachers' mathematical communication.

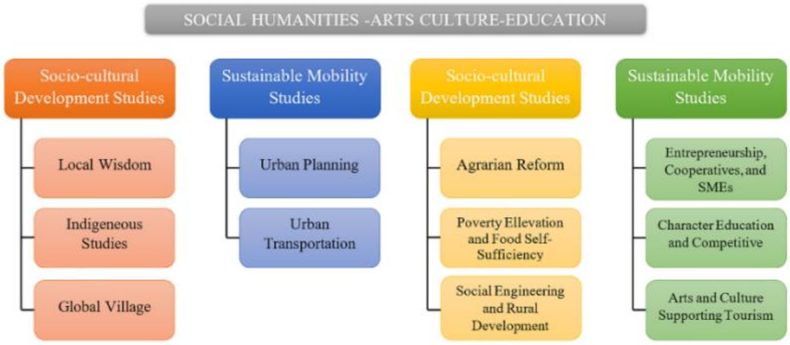


Fig. 3 Indonesian Research Focus in 2017-2045

The Indonesian National Research Master Plan (RIRN) document states that from 2017–2045, research in Indonesia will focus on the theme of social humanities–cultural arts education, as seen in Figure 3 above. Mathematical communication research, one of the sub-focuses of educational research, aims to explore the quality of competitive human resources, and in this vein, mathematical communication research needs to receive more attention.

2. Method

2.1 Research Design

This research was descriptive and qualitative in design. This is qualitative research because it does not use numerical data but uses narrative data or words. Specifically, the researchers analysed pre-service mathematics teachers' mathematical communication skills and classified them into three categories. A qualitative approach was used because the research explores a social phenomenon or process (Creswell, 2014). Research data are presented based on facts gathered in the field without manipulation (Sagala et al., 2019).

2.2 Participants

The study subjects were pre-service mathematics teachers from various regions of Indonesia. They are 105 pre-sevice mathematics teachers in total. The subjects were chosen because (1) they were willing to volunteer, (2) they employed mathematical communication in problem solving, and (3) they could implement think-aloud approaches when solving problems. The exclusion criteria for pre-service mathematics teachers were not being able to employ mathematical communication or not being able to implement think-aloud approaches in problem solving.

2.3 Instruments

The instruments used to collect data in this research were test instruments (non-routine tests), observation sheets, and interview guidelines. These instruments were developed based on the research objectives, the construction of mathematical problems (Yorulmaz et al., 2021), and the suitability of the discussion to the subject's ability to understand. Before the instrument was employed to collect data, all three instruments were validated by experts in mathematics education research and mathematical communication. The input from the validators was that the sentence structure needed to be simplified to be more easily understood by problem solvers, and images or illustrations needed to be added to make the problems more contextual. The participants did not receive any special treatment in obtaining the natural data. Figure 4 represents a non-routine problem used to classify pre-service mathematics teachers mathematical communication.

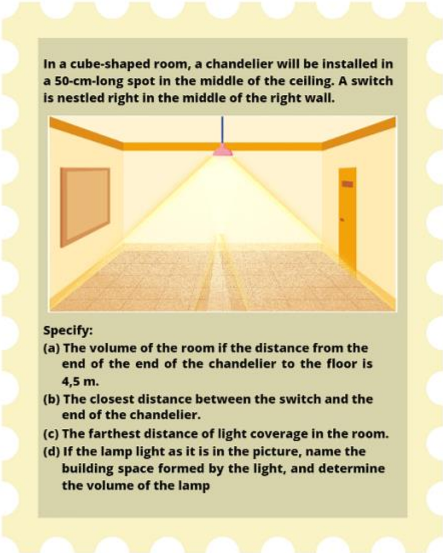


Fig. 4 Non-Routine Mathematical Problem

2.4 Indicators of Mathematics Communication

We adopted the mathematical communication indicators from the levels of mathematical communication specified by Uyen et al. (2021) in that the level of mathematical communication can be assessed based on three aspects: mathematical vocabulary, representation, and explanation. We then developed indicators for these three aspects, which were employed as benchmarks to classify pre-service mathematics teachers' mathematical communication. Table 1 lists the aspects and indicators of mathematical communication. In mathematics vocabulary, the two indicators are coded as MV1 and MV2. The representation aspect included the two indicators coded R1 and R2. The explanation aspect consisted of the two indicators coded E1 and E2. These codes were compiled to assist the researchers in conducting the data analysis.

Table 1. Aspects and Indicators of Mathematics Communication

Aspect	Indicator	Code
Mathematics Vocabulary	1. Able to understand the problem	MV1
	2. Able to change problems in the language of mathematics	MV2
Representation	1. Understand how to represent a problem	R1
	2. Able to represent the problem	R2
Explanation	1. Understand how to explain answers	E1
	2. Able to provide explanations of answers orally and in writing	E2

2.5 Data Collecting Procedure

Data were collected using the following procedure:

1. Researchers administered mathematical communication tests using non-routine problems. One by one, the subjects solved mathematical problems using the think-aloud technique. They continued to speak as they progressed along the mathematical problem-solving process to report the communication symptoms they experienced. Thinking aloud requires the subject to speak aloud when solving a problem or performing a task. This method has traditionally been applied in psychological and educational research on cognitive processes and knowledge acquisition in building knowledge-based computer systems. The think-aloud technique is frequently a novel source of knowledge about cognitive processes that yield precise information on ongoing thought processes during task performance (Jaspers et al., 2004). According to Olson et al. (1984), the think-aloud technique is one of the best ways to evaluate higher-order thinking processes (which involve working memory), and it can also be used to study individual differences in performing the same task. This method is suitable for critically exploring and capturing symptoms in qualitative research (Rankine, 2019). While the subjects were engaged in this process, the researcher made an audio-visual recording and wrote down some of the findings on the observation sheet.
2. Researchers examined pre-service mathematics teachers answer sheets, think-aloud recordings, and observation sheets to analyse the subjects' mathematical communication. Furthermore, the researcher conducted interviews with the subjects until complete and comprehensive data for categorising mathematical communication were obtained.
3. The researcher compared answer sheets, think-aloud recordings, observation sheets, and interview results to classify the levels of mathematical communication.

2.6 Data Analysis

In data analysis, researchers collect and sort data into several categories, classify additional information, and then arrive at a classification system. This process is based on the constant comparative procedure (CCP) outlined by (Creswell, 2012) and (Glaser, 1992). The CCP was used as an inductive data analysis procedure to generalise and classify mathematical communication categories by comparing data from different techniques. From incident to incident and category to category, this has also been implemented in previous studies (Kholid, Sa'Dijah et al., 2022; Kholid, Swastika et al., 2022; Sa'dijah et al., 2020). An illustration of the CCP is presented in Figure 5.

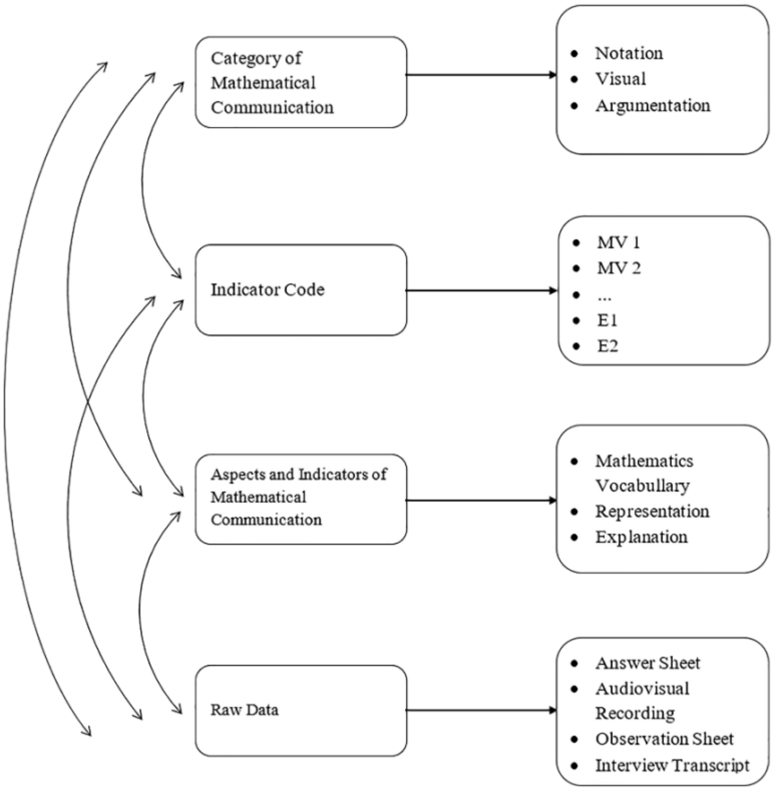


Fig. 5 Constant Comparative Procedure

This research implemented CCP by changing the raw data from pre-service mathematics teachers answer sheets, observation sheets, think-aloud transcripts, and interview transcripts into mathematical communication classifications. Data were obtained from several subjects using various methods. Then, the researcher constantly compared mathematical communication indicators and categories through the CCP process. In order to develop mathematical communication classification proofs and eliminate duplication, this was done.

2.7 Validity of Data

With the help of the source triangulation method, the data's validity was evaluated. Triangulation is applied when data collection uses various techniques or methods, such as tests, interviews, and observations. The researcher collected data from various participants to obtain a valid and consistent mathematical classification of communication.

3. Results and Discussion

After collecting and analysing the data, three categories of mathematical communication were identified. Each category presents data from a different group of pre-service mathematics teachers: S-1 are pre-service mathematics teachers in the notation category, S-2 are pre-service mathematics teachers in the visual category, and S-3 are pre-service mathematics teachers in the argumentation category. Data from all three sets of subjects showed early symptoms of data credibility. Table 2 shows the distribution of subjects in each category.

Table 2. Distribution of the Subject

Category	Sum	Percentage
Notation	42	40%
Visual	31	29,52%
Argumentation	32	30,48%
Sum	105	100%

3.1 Category 1:Notation

S-1 demonstrated the **MV1** indicator by repeatedly scanning and writing vital information to understand a problem. Before answering the question, S-1 described a cube by referring to the chandelier's location, the switch's location, and the chandelier's length $= 0.5$ m and wrote that the distance of the chandelier to the floor was 4.5 m. At first, S-1 had written that the side of the cube was 50 cm, which meant that S-1 had experienced errors in reading or understanding problems, but S-1 could correct these errors and continue solving problems. The **MV2** indicator was demonstrated when S-1 wrote problems in mathematical language. From the data obtained, S-1 could answer all questions using mathematical notations. In determining the answer to problem (a), S-1 wrote that $\text{Room Volume} = s \times s \times s = 5 \times 5 \times 5 = 125\ \text{m}^3$. This showed that S-1 could conclude that the length of the side of the room was 5 m after understanding the Problem; however, S-1 could also write the answer in mathematical notation. S-1's answer to Problem (a) is presented in Figure 6.

When answering Problem (b), S-1 could obtain the length of the sides of the triangle, but it seemed that S-1 was confused in describing the triangle for which he already knew the length of the sides. Nevertheless, S-1 could soon describe a triangle with just the right-side length. After describing the triangle as an aid, S-1 solved the Problem and obtained precise results regarding the closest distance between the switch and the lamp. In solving Problem (b), S-1 could again represent the answer in mathematical notation. S-1's answer to Problem (b) is presented in Figure 7.

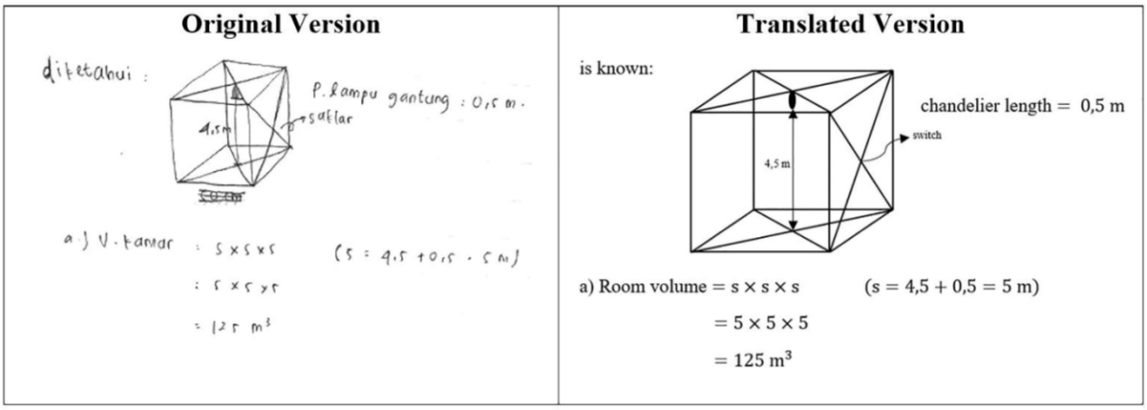


Fig. 6 S-1 Worksheet for Solving Problem (a)

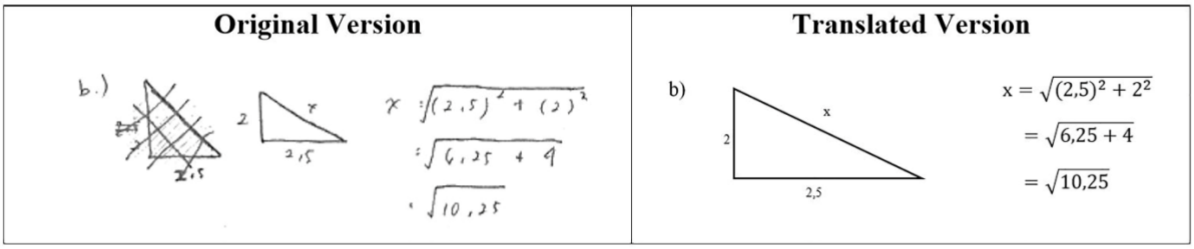


Fig. 7 S-1 Worksheet for Solving Problem (b)

Similar to Problem (b), in solving Problem (c), S-1 could describe a triangle before determining the answer. However, it looked like S-1 was having trouble understanding the Problem. Although S-1 represented the answer with mathematical notation, the answer was incorrect. In determining the farthest distance of the lamplight, S-1 took two steps, both of which used the Pythagorean theorem. In the first step, S-1 wrote the length as x, and then in the second step, S-1 entered the value for x and looked for x as a new variable valued at 5.5. However, after the analysis, the first and second steps could not be applied in a cube, which meant that S-1's answer was incorrect. S-1's answer to Problem (c) is presented in Figure 8.

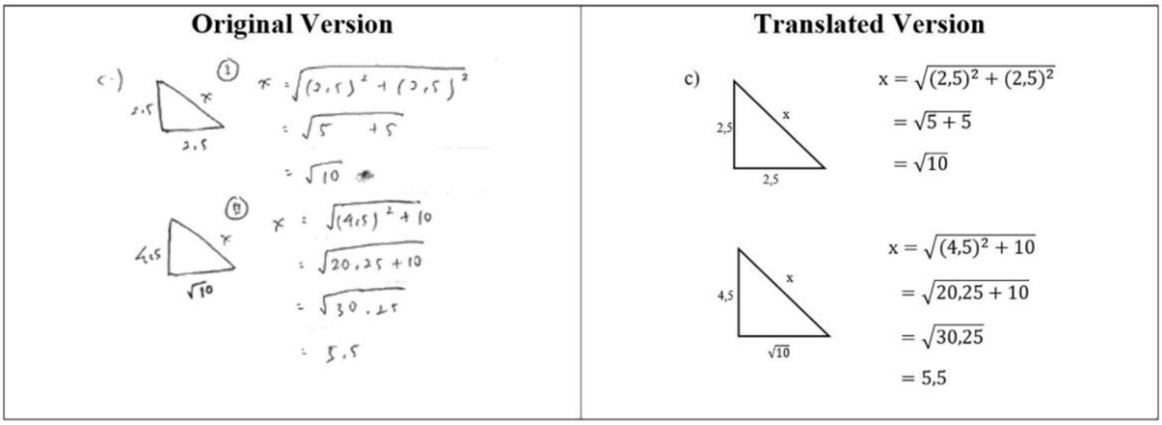


Fig. 9 S-1 Worksheet for Solving Problem (c)

Supposedly, in solving this problem you only need one step, namely drawing an isosceles triangle like the figure 9 as an example. Next, we can enter the length based on the information obtained previously. Once the length of the side is known, we can find the farthest reach of light using the Pythagorean theorem, namely $x = \sqrt{(4{,}5)^2 + (2{,}5\sqrt{2})^2} = \sqrt{20{,}25 + 12{,}5} = \sqrt{32{,}75} = 5{,}72$.

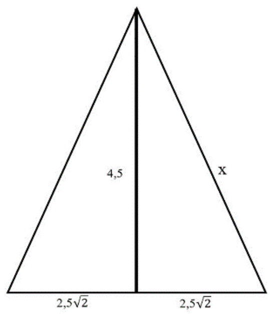


Fig. 9 Isosceles triangle for help working on problems (c)

In solving the last Problem or Problem (d), S-1 understood the form of the building in question, namely a rectangular pyramid space. Then, S-1 could write the formula of the rectangular pyramid's volume and precisely determine the sides of the pyramid. S-1 wrote that Volume $= \frac{1}{3} \times$ base area $\times$ height $= \frac{1}{3} \times 5 \times 5 \times 4.5 = 37.5$ m. S-1 was then able to calculate the multiplication result and found the correct volume of the space, but S-1 forgot the unit part of the volume. Specifically, S-1 only wrote m as a volume unit instead of m^3. This makes the answer from S-1 arguably less precise, even though it is only a unit issue. S-1's answer to Problem (d) is presented in Figure 10.

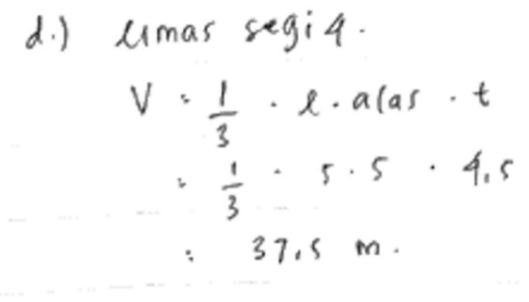
Original Version	Translated Version
	d. Rectangular Pyramid $\text{Volume} = \frac{1}{3} \times \text{base are} \times \text{height}$ $= \frac{1}{3} \times 5 \times 5 \times 4{,}5$ $= 37{,}5\ \text{m}^3$

Fig. 10 S-1 Worksheet for Solving Problem (d)

The data from S-1's process show that in solving non-routine problems, S-1 relied on mathematical communication in the notation category. This was shown when solving Problem (a) using the formula $\text{Room Volume} = s \times s \times s$. Furthermore, in solving Problem (b) and Problem (c), S-1 used the variable x. In addition, S-1 solved Problem (d) by using the formula $\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height}$.

3.2 Category 2: Visual

S-2 displayed the **R1** and **R2** indicators by representing the Problem as an image. To facilitate problem solving, S-2 first made a complete cube drawing, which was then broken down into right triangles to obtain answers using the Pythagorean theorem.

In solving Problem (a), S-2 wrote that the length of the side of the cube $= 0.5\ \text{m} + 4.5\ \text{m} = 5\ \text{m}$. S-2 obtained this information to determine the length of the side of the room or the length of the side of the cube. Then, S-2 calculated the volume of the room by writing the volume of the cube $= s \times s \times s = 5 \times 5 \times 5 = 125\ \text{m}$. From the answer, it can be seen that S-2 was correctly able to determine the volume of the room, but there was still an error in writing the unit. S-2's answer to Problem (a) is presented in Figure 11.

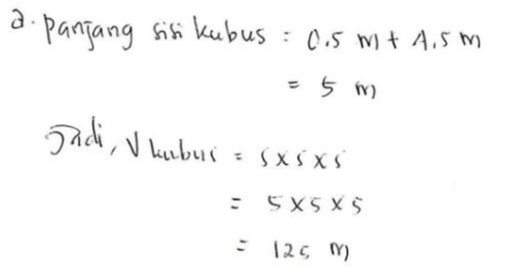
Original Version	Translate Version
	a. Side length of the cube $= 0{,}5\ \text{m} + 4{,}5\ \text{m}$ $= 5\ \text{m}$ So, cube volume $= s \times s \times s$ $= 5 \times 5 \times 5$ $= 125\ \text{m}$

Fig. 11 S-2 Worksheet for Solving Problem (a)

When solving Problem (b), S-2 visualised by drawing a complete cube with a triangle to help facilitate the work. The image created by S-2 was then broken down again into two right triangles that were 2.5 high and 2.5 long. S-2 used the Pythagorean theorem with the right triangle described earlier to determine the shortest distance between the switches and lamp ends. However, there was a misconception that the distance sought by S-2 was not the closest because the S-2 used the upper end of the chandelier. S-2 needed to use the chandelier's lower end instead of the chandelier's top end to find the closest distance. S-2's answer to Problem (b) is presented in Figure 12.

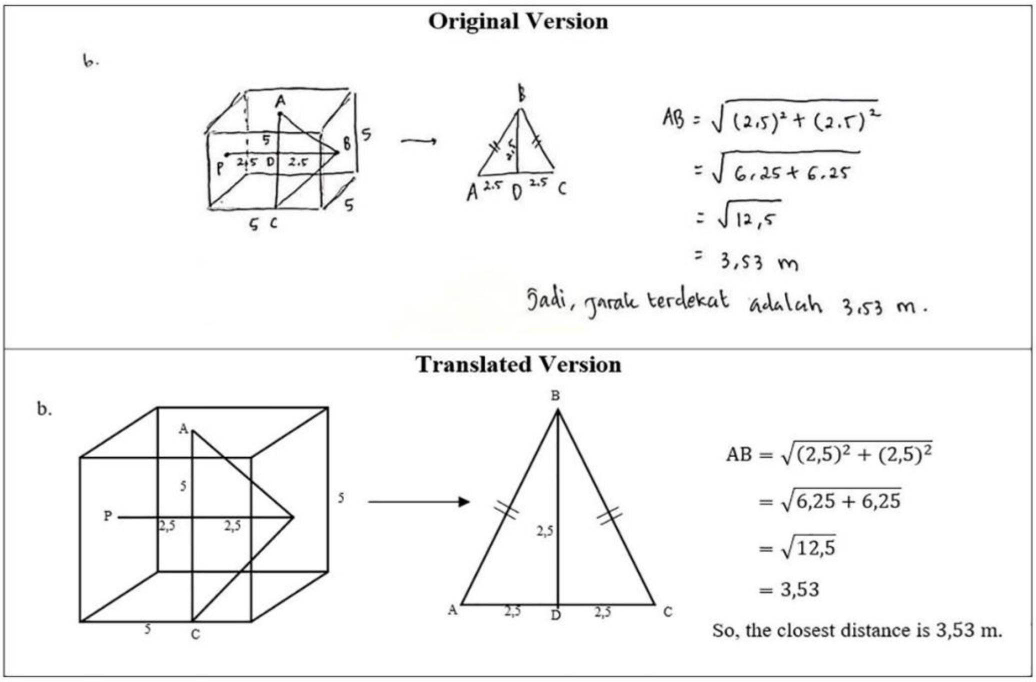


Fig. 12 S-2 Worksheet for Solving Problem (b)

Similar to Problem (b), in solving Problem (c), S-2 also engaged in visualisation by describing a complete cube with a rectangular pyramid space. Then, to find the farthest distance from the lamplight, S-2 broke the image down into two right triangles, each with a height of 4.5 and a base length of $2.5\sqrt{2}$. Then, to find the farthest distance of the lamplight, S-2 used the Pythagorean theorem with the right triangle described. S-2 determined the longest range of the light bulbs to be 5.72 m. S-2's answer to Problem (c) is presented in Figure 13.

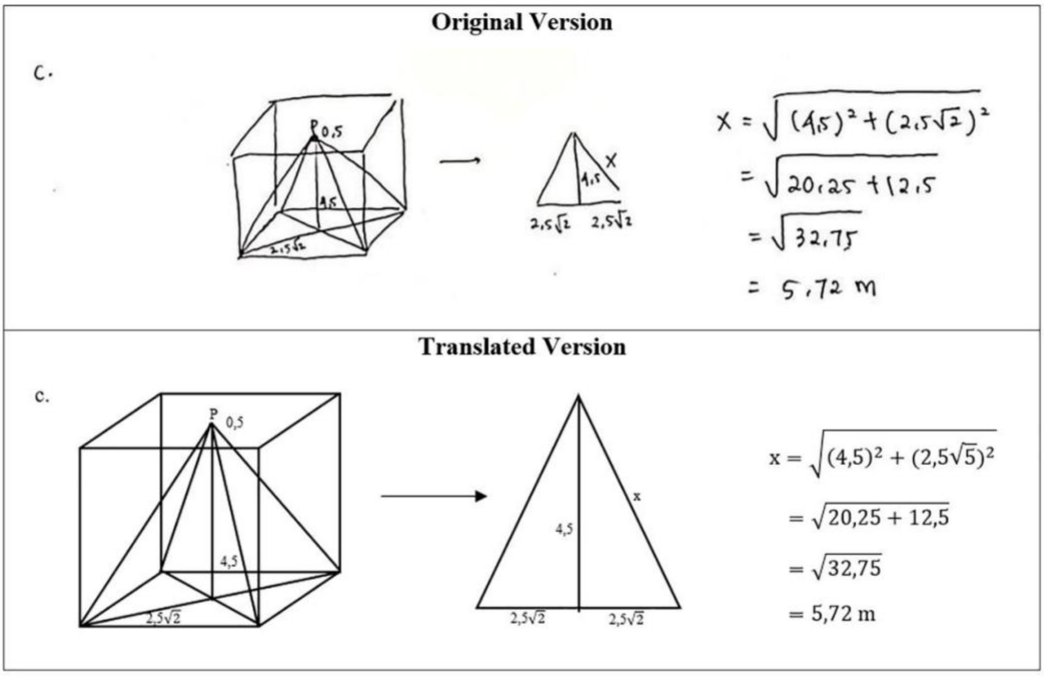


Fig. 13 S-2 Worksheet for Solving Problem (c)

In solving Problem (d), S-2 wrote that the space formed was a rectangular pyramid. Then, S-2 determined the volume of the wake by writing Volume $= \frac{1}{3} \times \text{base area} \times \text{height} = \frac{1}{3} \times 5 \times 5 \times 4{,}5 = 37.5$ m. Similar to Problem (a), S-2 was still inaccurate in writing the unit of volume, even though the value was correct. S-2's answer to Problem (d) is presented in Figure 14.

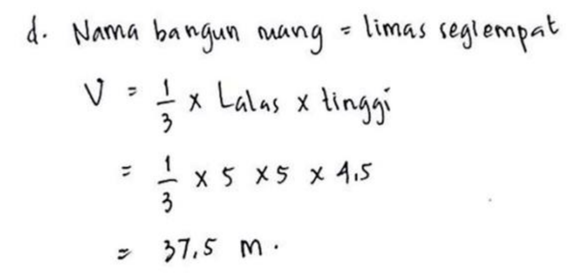
Original Version	Translated Version
	d. The name of the geometric shape = rectangular pyramid $V = \frac{1}{3} \times \text{base area} \times \text{height}$ $= \frac{1}{3} \times 5 \times 5 \times 4{,}5$ $= 37{,}5$ metre

Fig. 14 S-2 Worksheet for Solving Problem (d)

S-2's data show that in solving non-routine problems, S-2 used visual mathematical communication. This was especially evident when solving problems (b) and (c). S-2 visualised the problems by fully describing the information obtained from the Problem itself to help devise a solution.

3.3 Category 3: Argumentation

S-3 demonstrated the E1 indicator by scanning the question and discerning small details to find a way to explain the answer. Before answering the question, S-3 described a cube, which included the lamp's length and the distance to the floor. Then, the E2 indicator became evident when S-3 answered all four questions with clear and logical arguments.

In solving Problem (a), S-3 responded by presenting an argument regarding how the room's length/height/width could be obtained from the length of the lamp tip to the floor and the length of the lamp. S-3 concluded that the length of the chamber side was 5 m. S-3 then described the formula that would be used to find the volume of the room, namely Volume $= s \times s \times s$. S-3's answer to Problem (a) is presented in Figure 15.

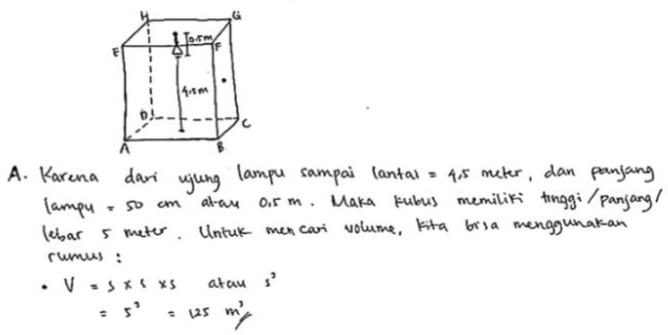
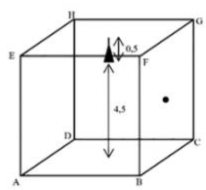
Original Version	Translated Version
	 A. This is because from the tip of the lamp to the floor = 4.5 metres, and the length of the lamp = 50 cm or 0.5 m. Then, the cube has a height/length/width of 5 metres. To find the volume, we can use the following formula: Volume $= s \times s \times s$ or s^3 $= 5^3 = 125\ m^3$

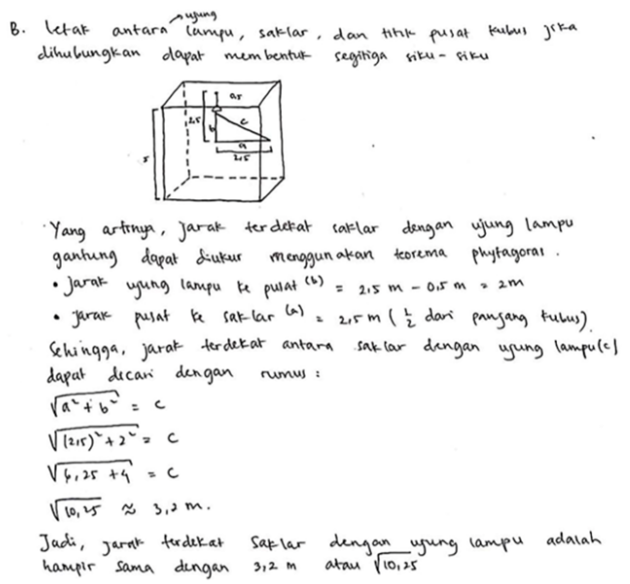
Fig. 15 S-3 Worksheet for Solving Problem (a)

In solving Problem (b), S-3 responded by arguing that the location between the end of the lamp, the switch, and the center point of the cube, if connected, would form a right triangle. To facilitate the work, S-3 also provided an image of the triangle in a cube. Then, S-3 explained that the switch's closest distance to the chandelier's end could be found using the Pythagorean theorem. S-3 used the variable a to refer to the distance of the cube's center to the switch, and the distance of the lamp's tip to the cube's center was represented by variable b. The Pythagorean theorem was then used to obtain the value of c as the closest distance between the switch and the end of the lamp. S-3 concluded that the closest distance of the switch to the tip of the lamp was 3.2 m or $\sqrt{10,25}$. S-3's answer to Problem (b) is presented in Figure 16.

When solving Problem (c), S-3 initially made a drawing to make the work easier. The image showed that the chandelier was right on the middle side of the room's ceiling, and S-3 concluded that the farthest range of light was in the corner of the lower side of the floor, which was represented by ∠A, ∠B, ∠C, and ∠D. S-3 explained that, if one angle was chosen, the farthest distance of the lamp could be determined because a right triangle would be formed. The farthest length of light could then be found using the Pythagorean theorem. At first, S-3 determined the length of the side of the chamber and the length of the diagonal half of the plane. The data allowed S-3 to determine the farthest distance from the lamp light, which was $\frac{\sqrt{131}}{2}$, almost equal to 5.72276. S-3's answer to Problem (c) is presented in Figure 17.

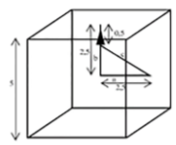
S-3's solution to Problem (d) also relied on a picture, which S-3 briefly explained by referring to how the building space formed from the lamp's light was a rectangular pyramid, with the top being the lower end of the lamp and the four corners of the base being the corners of the lower room. Then, S-3 wrote the formula to find the volume of the rectangular pyramid as Volume $= \frac{1}{3} \times$ base area $\times$ height $= \frac{1}{3} \times (5 \times 5) \times 4{,}5 = \frac{1}{3} \times 25 \times 4{,}5 = 37{,}5\ \mathrm{m}^3$. S-3's answer to Problem (d) is presented in Figure 18.

Original Version



Translated Version

B. When connected, the location between the end of the lamp, the switch, and the centre point of the cube can form a right triangle.



This means that the switch's closest distance to the chandelier's end can be measured using the Pythagorean theorem.

- The length of the lamp end to the centre (b) = 2.5 m − 0.5 m = 2 m.

Centre distance to switch (a) = 2.5 m ($\frac{1}{2}$ of cube length)

Thus, the closest distance between the switch and the tip of the lamp (c) can be found by the following formula:

$\sqrt{a^2 + b^2} = c$

$\sqrt{(2.5)^2 + 2^2} = c$

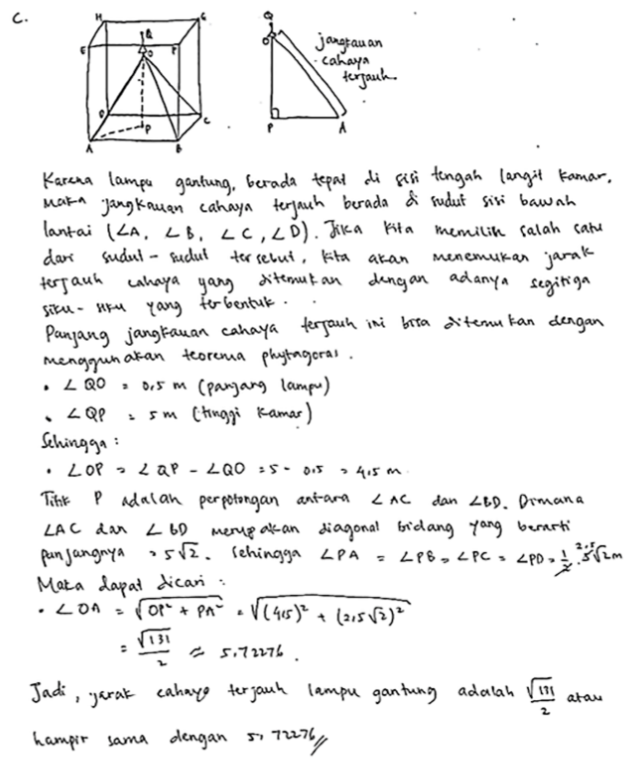
$\sqrt{6.25 + 4} = c$

$\sqrt{10.25} \approx 3{,}2$ m

So, the closest distance of the switch to the tip of the lamp is almost the same as 3.2 m or $\sqrt{10.25}$

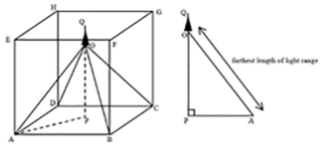
Fig. 16 S-3 Worksheet for Solving Problem (b)

Original Version



Translated Version

C.



Because the chandelier is right in the centre of the room's ceiling, the farthest range of light is in the corner below the floor (∠A, ∠B, ∠C, and ∠D). If we choose one of these angles, we will find the farthest distance of light in the presence of a right triangle. This farthest length of the light range can be found using the Pythagorean theorem.

- $\angle$ QO = 0.5 m (lamp length)
- $\angle$ QP = 5 m (room height)

So that:

- $\angle OP = \angle QP - \angle QO = 5 - 0{,}5 = 4.5$ m

Point P is the intersection between ∠ AC and ∠ BD, which means its length = $5\sqrt{2}$. So,

$\angle PA = \angle PB = \angle PC = \angle PD = \frac{1}{2} \times 5\sqrt{2}$ m

Then, the following can be calculated:

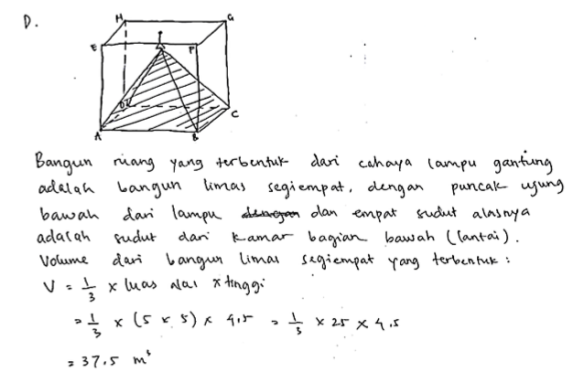
- $\angle OA = \sqrt{OP^2 + PA^2} = \sqrt{(4.5) + (2.5\sqrt{2})^2}$

$= \frac{\sqrt{131}}{2} \approx 5.72276$

So, the farthest light distance of the chandelier is $\frac{\sqrt{131}}{2}$ or almost the same as 5.72276.

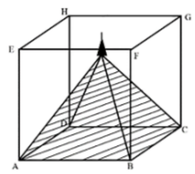
Fig. 17 S-3 Worksheet for Solving Problem (c)

Original Version



Translated Version

D.



Building a space formed from the chandelier's light allows for a rectangular pyramid to be created with the bottom end of the lamp, and the four corners of the base are the lower corners of the room (floor). The volume of the formed rectangular pyramid can be calculated as follows:

$V = \frac{1}{3} \times \text{base area} \times \text{height}$

$= \frac{1}{3} \times (5 \times 5) \times 4{,}5 = \frac{1}{3} \times 25 \times 4{,}5$

$= 37{,}5 \text{ m}^3$

Fig. 18 S-3 Worksheet for Solving Problem (d)

S-3's solution shows that in solving non-routine problems, S-3 relied on mathematical communication in the argumentation category. For each Problem, S-3 always explained the steps toward the solution through logical and systematic arguments.

3.4 Discussion

Referring to the results of the data analysis above, S-1 can be categorised as representing pre-service mathematics teachers who tend to use the notation style of mathematical communication. This is because in solving problems, this group of pre-service mathematics teachers represents mathematical problems through notation and symbols. According to Rohid et al. (2019), pre-service mathematics teachers who convert math problems into symbols and notations significantly improve their ability to solve math problems. According to Wijayanto et al. (2018), this aligns with the Ministry of National Education's learning objectives, which state that pre-service mathematics teachers should learn how to express ideas using symbols to clarify situations or problems. In addition, this is consistent with the idea of O'Halloran (2006) that one type of mathematical discourse involves mathematical symbolism.

S-2 represents pre-service mathematics teachers with visual and mathematical communication tendencies. Specifically, these pre-service mathematics teachers visualise mathematical problems with the use of pictures. In solving mathematical problems, pre-service mathematics teachers need accurate visualization to realize problem solving (Kholid et al., 2022). According to Syahri (2017), verbal communication skills can be displayed in the form of verbal expressions and explanations of mathematical ideas, such as speaking, listening, and discussing, while written communication skills can emerge in the form of pictures, tables, graphs, questions, and other types of answers in written form. By making a picture, pre-service mathematics teachers can more easily understand a given problem (NCTM, 2000). Greenes and Schulman (1996) elaborated on this point through the idea that one mathematical communication indicator is using mathematical representations, such as images, to represent mathematical information. Visualisation can therefore be used as a criterion of mathematical communication ability (Sumaji et al., 2020).

The last category consists of pre-service mathematics teachers who engage in the argumentation type of mathematical communication. In solving problems, S-3 presented arguments logically and systematically. According to the NCTM (2000), communication is fundamental in mathematics because it allows pre-service mathematics teachers to present mathematical ideas orally or in writing. Pre-service mathematics teachers need to know the meaning of mathematical vocabulary in order to understand better and communicate mathematical ideas (Gay, 2008). This follows the opinion of Silver et al., (1990) who state that mathematical communication is more helpful, as individuals can explain an idea in more detail through this approach. According to Kholid et al. (2020), differences in cognitive style affect pre-service mathematics teachers ability to present arguments logically and systematically.

4. Conclusion

There are three categories of pre-service mathematics teachers mathematical communication, namely the notation category, the visual category, and the argumentation category. The notation category is characterised by the tendency of pre-service mathematics teachers to represent mathematical problems in the form of symbols. In the visual category, pre-service mathematics teachers visualise problems by describing the information they contain. The third category is argumentation, in which pre-service mathematics teachers answer questions by presenting logical and systematic arguments.

5. Suggestions

The findings presented here show that there are three categories of mathematical communication. However, this research is limited to solving three-dimensional geometry problems. There may be different research results when using different types of problem subjects. Further research is needed to focus on the factors influencing mathematical communication and its improvement through the development of IT-based learning media.

6. Acknowledgements

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