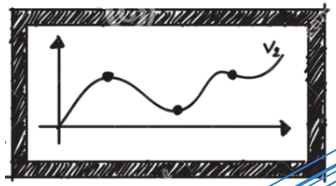
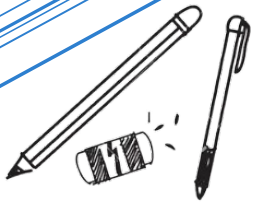
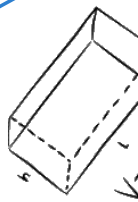
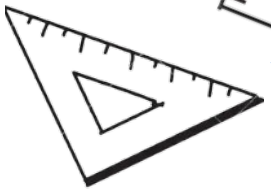
MASTERING DERATIVES

SIMPLIFY THE COMPLEX!!









$$\frac{du}{dx} = \frac{du}{dy} = \frac{dy}{dx}$$

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$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

MASTERING DERIVATIVES

SIMPLIFY THE COMPLEX!

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PREFACE

This e-book, Mastering Derivatives: Simplify the Complex! aimed to help students in Calculus subject. Targeted users for this e-book are students who take foundation course. Mathematical tips and formulas will be placed in accordance with the subtopics whilst each question will be displayed based on the syllabus carried out during the lesson. At the end of each topic, targeted students should meet up with the lecturer to discuss over the solution of mathematics problem. With the existence of this e-book, hopefully it will be beneficial and give positive impact towards teaching and learning for students and lecturers.

1.0 BASIC OF DIFFERENTIATION

FORMULA

$$y = x^n = n(x^{n-1})$$

$$y = (x^n + 1)^m = m(x^n + 1)^{m-1}.\frac{d}{dx}(x^n + 1)$$

$$y = e^x = e^x.\frac{d}{dx}(x)$$

$$y = \ln x = \frac{1}{x}.\frac{d}{dx}(x)$$

$$PRODUCT\ RULE > \frac{dy}{dx} = u'v + v'u$$

$$QUOTIENT\ RULE > \frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

QUESTIONS

1. $y = (2x^4 + 1)^6$

2. $y = (3x^2 - 7x)^4$

3. $y = \sqrt[3]{4 - 6x}$

4. $y = \frac{1}{5x^2 - 8}$

5. $y = \frac{6}{(7x+8)^5}$

6. $y = (x^2 - x + 2)(x^2 - 3)$

7. $y = 4x(x^2 - 8)$

8. $y = 4x(x^2 - 8)$

9. $y = (x + 4)^2 - 5x$

10. $y = (1 - 2x)^3(3x + 1)^4$

11. $y = (x + 3)^{\frac{3}{2}}(x - 2)^{\frac{1}{4}}$

12. $y = \frac{x^2}{1+x}$

13. $y = \ln(\frac{6x}{1-2x^2})$

14. $y = \frac{1}{7(2x+9)^3}$

15. $y = x^3 \ln(5x - 2)$

16. $y = x(x^2 - 7)$

SOLUTIONS

1. $\boldsymbol{y = (2x^4 + 1)^6}$

$y' = 6(2x^4 + 1)^5 \cdot \frac{d}{dx}(2x^4 + 1)$

$= 6(2x^4 + 1)^5 \cdot 8x^3$

$y' = 48x^3(2x^4 + 1)^5$

2. $\boldsymbol{y = (3x^2 - 7x)^4}$

$y' = 4(3x^2 - 7x)^3 \cdot \frac{d}{dx}(3x^2 - 7x)$

$= 4(3x^2 - 7x)^3 \cdot (6x - 7)$

$y' = 4(6x - 7)(3x^2 - 7x)^3$

3. $\boldsymbol{y = \sqrt[3]{4 - 6x}}$

$y = (4 - 6x)^{\frac{1}{3}}$

$y' = \frac{1}{3}(4 - 6x)^{-\frac{2}{3}} \cdot \frac{d}{dx}(4 - 6x)$

$= \frac{1}{3}(4 - 6x)^{-\frac{2}{3}} \cdot (-6)$

$y' = -\frac{2}{(4-6x)^{\frac{2}{3}}}$

4. $\boldsymbol{y = \frac{1}{5x^2 - 8}}$

$y = (5x^2 - 8)^{-1}$

$y' = -1(5x^2 - 8)^{-2} \cdot \frac{d}{dx}(5x^2 - 8)$

$= -1(5x^2 - 8)^{-2} \cdot (10x)$

$y' = -\frac{10}{(5x^2-8)^2}$

5. $\boldsymbol{y = \frac{6}{(7x+8)^5}}$

$y = 6(7x + 8)^{-5}$

$y' = -30(7x + 8)^{-6} \cdot \frac{d}{dx}(7x + 8)$

$= -30(7x + 8)^{-6} \cdot (7)$

$= -210(7x + 8)^{-6}$

$y' = -\frac{210}{(7x+8)^6}$

6. $\boldsymbol{y = (x^2 - x + 2)(x^2 - 3)}$

- EXPAND

$y = x^4 - 3x^2 - x^3 + 3x + 2x^2 - 6$

$= x^4 - x^2 - x^3 + 3x - 6$

- DIFFERENTIATE

$y' = 4x^3 - 2x - 3x^2 + 3$

7. $\boldsymbol{y = 4x(x^2 - 8)}$ $y = 4x^3 - 32x$ $y' = 12x^2 - 32$	**8.** $\boldsymbol{y = \sqrt[3]{x^2 - 3x}}$ $y = (x^2 - 3x)^{\frac{1}{3}}$ $y' = \frac{1}{3}(x^2 - 3x)^{-\frac{2}{3}} \cdot \frac{d}{dx}(x^2 - 3x)$ $y' = \frac{1}{3}(x^2 - 3x) \cdot (2x - 3)$

9. $\boldsymbol{y = (x+4)^2 - 5x}$

$y = (x+4)(x+4) - 5x$

$= x^2 + 8x + 16 - 5x$

$= x^2 + 3x + 16$

$y' = 2x + 3$

10. $\boldsymbol{y = (1-2x)^3(3x+1)^4}$

$u = (1-2x)^3$ $\qquad$ $v = (3x+1)^4$

$u' = 3(1-2x)^2.\frac{d}{dx}(1-2x)$ $\qquad$ $v' = 4(3x+1)^3.\frac{d}{dx}(3x+1)$

$= 3(1-2x)^2.(-2)$ $\qquad$ $= 4(3x+1)^3.(3)$

$= -6(1-2x)^2$ $\qquad$ $= 12(3x+1)^3$

- $\boldsymbol{uv' + vu'}$

$y' = (1-2x)^3.12(3x+1)^3 + (3x+1)^4.-6(1-2x)^2$

$= 12(3x+1)^3(1-2x)^3 - 6(3x+1)^4(1-2x)^2$

$= 6(3x+1)^3(1-2x)^2[2(1-2x) - 1(3x+1)]$

$= 6(3x+1)^3(1-2x)^2[2-4x-3x-1]$

$y' = 6(3x+1)^3(1-2x)^2[1-7x]$

11. $\boldsymbol{y = (x+3)^{\frac{3}{2}}(x-2)^{\frac{1}{4}}}$

$u = (x+3)^{\frac{3}{2}} \qquad v = (x-2)^{\frac{1}{4}}$

$u' = \frac{3}{2}(x+3)^{\frac{1}{2}} \qquad v' = \frac{1}{4}(x-2)^{-\frac{3}{4}}$

$u' = \frac{3\sqrt{x+3}}{2} \qquad v' = \frac{1}{4(\sqrt[4]{(x-2)^3})}$

- $\boldsymbol{uv' + vu'}$

$= (x+3)^{\frac{3}{2}}(\frac{1}{4(\sqrt[4]{(x-2)^3})}) + (x-2)^{\frac{1}{4}}(\frac{3\sqrt{x+3}}{2})$

$y' = \frac{\sqrt{(x+3)^3}}{4(\sqrt[4]{x-2^3})} + \frac{\sqrt[4]{(x-2)}\,.\,3\sqrt{x+3}}{2}$

12. $\boldsymbol{y = \frac{x^2}{1+x}}$

$u + x^2$ $\qquad$ $v = 1 + x$

$\frac{du}{dx} = 2x$ $\qquad$ $v' = 1$

- $\boldsymbol{\frac{vu' - uv'}{v^2}}$

$$y' = \frac{(1+x)(2x) - x^2(1)}{(1+x)^2}$$

$$= \frac{2x + 2x^2 - x^2}{(1+x)^2}$$

$$y' = \frac{2x + x^2}{(1+x)^2}$$

13. $\boldsymbol{y = \ln\left(\frac{6x}{1-2x^2}\right)}$

$$y = \ln(6x) - \ln(1 - 2x^2)$$

$$y' = \frac{1}{6x}.\frac{d}{dx}(6x) - \frac{1}{1-2x^2}.\frac{d}{dx}(1 - 2x^2)$$

$$= \frac{1}{6x}.(6) - \frac{1}{1-2x^2}(4x)$$

$$y' = \frac{1}{x} + \frac{4x}{1-2x^2}$$

14. $\boldsymbol{y = \frac{1}{7(2x+9)^3}}$

$$y = \frac{1}{7}(2x + 9)^{-3}$$

$$y' = \frac{1}{7}(-3)(2x + 9)^{-4}(2)$$

$$= -\frac{6}{7}(2x + 9)^{-4}$$

$$y' = -\frac{6}{7(2x+9)^{-4}}$$

15. $\boldsymbol{y = x^3 \ln(5x - 2)}$

$u = x^3$ $\qquad$ $v = \ln(5x - 2)$

$u' = 3x^2$ $\qquad$ $v' = \frac{1}{5x-2} \cdot \frac{d}{dx}(5x - 2)$

$v' = \frac{1}{5x-2}(5)$

$v' = \frac{5}{5x-2}$

- $\boldsymbol{uv' + vu'}$

$y' = (x^3)\left(\frac{5}{5x-2}\right) + \ln(5x - 2)\,(3x^2)$

$y' = \frac{5x^3}{5x-2} + 3x^2 \ln 5x - 2$

16. $\boldsymbol{y = x(x^2 - 7)}$

$y = x^3 - 7x$

$\boldsymbol{y' = 3x^2 - 7}$

- **OR**

$u = x \qquad v = x^2 - 7$

$u' = 1 \qquad v' = 2x$

- $\boldsymbol{uv' + vu'}$

$y' = x(2x) + (x^2 - 7)(1)$

$= 2x^2 + (x^2 - 7)$

$= 2x^2 + x^2 - 7$

$y' = 3x^2 - 7$

2.0 DIFFERENTIATION OF TRIGONOMETRY

FORMULA

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\text{cosec}\, x) = -cosec\ x \cot x$$

QUESTIONS

1. $y = e^{5-3x}$

2. $y = e^{\sqrt[3]{x}}$

3. $y = \ln(2x - x^5)$

4. $y = \ln\frac{2x^4}{x^2+4}$

5. $y = \cos(6x^2 - 5x)$

6. $y = \sec(4x + 8)$

7. $y = \tan(2x^2 + 7)$

8. $y = x^5(2 - 4x)^7$

9. $y = \frac{6}{\sqrt{1-2x^3}}$

10. $y = \sqrt[3]{x^2 - 8}$

11. $y = sin^2(3x + 2)$

12. $y = \frac{x^3}{\sqrt{3x-5}}$

13. $y = x^5 \ln(3x - 2)$

14. $y = \frac{1}{6(7x+8)^3}$

15. $y = \sqrt{3x^3 + 6x}$

16. $y = \cos(\frac{6-3x}{6+3x})$

SOLUTIONS

1. $y = e^{5-3x}$

$$y' = e^{5-3x} \cdot \frac{d}{dx}(5 - 3x)$$

$$= e^{5-3x} \cdot -3$$

$$y' = -3e^{5-3x}$$

2. $\boldsymbol{y = e^{\sqrt[3]{x}}}$

$y = e^{x^{\frac{1}{3}}}$

$y' = e^{x^{\frac{1}{3}}} \cdot \frac{d}{dx}(x^{\frac{1}{3}})$

$= e^{x^{\frac{1}{3}}} \cdot \frac{1}{3}x^{-\frac{2}{3}}$

$= e^{\sqrt[3]{x}} \cdot \frac{1}{3} \cdot \frac{1}{(\sqrt[3]{x})^2}$ $\qquad y' = \frac{e^{\sqrt[3]{x}}}{3(\sqrt[3]{x})^2}$

3. $\boldsymbol{y = \ln(2x - x^5)}$

$$y' = \frac{1}{2x - x^5} \cdot \frac{d}{dx}(2x - x^5)$$

$$= \frac{1}{2x - x^5} \cdot (2 - 5x^4)$$

$$y' = \frac{2 - 5x^4}{2x - x^5}$$

4. $\boldsymbol{y = \ln \frac{2x^4}{x^2+4}}$

$$y = \ln(2x^3) - \ln(x^2 + 4)$$

$$y' = \frac{1}{2x^3} \cdot \frac{d}{dx}(2x^3) - \frac{1}{x^2 + 4} \cdot \frac{d}{dx}(x^2 + 4)$$

$$= \frac{1}{2x^3} \cdot (6x^2) - \frac{1}{x^2+4} \cdot (2x + 0)$$

$$= \frac{6x^2}{2x^3} - \frac{2x}{x^2+4}$$

$$y' = \frac{3}{x} - \frac{2x}{x^2+4}$$

5. $\boldsymbol{y = \cos(6x^2 - 5x)}$

$y' = -\sin(6x^2 - 5x) \cdot \frac{d}{dx}(6x^2 - 5x)$

$= -\sin(6x^2 - 5x).(12x - 5)$

$y' = -(12x - 5)\ \sin(6x^2 - 5x)$

6. $\boldsymbol{y = \sec(4x + 8)}$

$y' = \sec(4x + 8)\tan(4x + 8).\frac{d}{dx}(4x + 8)$

$= \sec(4x + 8)\tan(4x + 8).(4)$

$y' = 4\sec(4x + 8)\tan(4x + 8)$

7. $\boldsymbol{y = \tan(2x^2 + 7)}$

$$y' = sec^2(2x^2 + 7).\frac{d}{dx}(2x^2 + 7)$$

$$= sec^2(2x^2 + 7).(4x)$$

$$y' = 4x\sec^2(2x^2 + 7)$$

8. $\boldsymbol{y = x^5(2 - 4x)^7}$

$u = x^5 \qquad v = (2 - 4x)^7$

$u' = 5x^4 \qquad v' = 7(2 - 4x)^6 . \frac{d}{dx}(2 - 4x)$

$= 7(2 - 4x)^6 . (-4)$

$= -28(2 - 4x)^6$

➢ $\boldsymbol{uv' + vu'}$

$= (x^5).[-28(2 - 4x)^6] + (2 - 4x)^7(5x^4)$

$= -28x^5(2 - 4x)^6 + 5x^4(2 - 4x)^7$

➢ **FACTORIZE**

$= x^4(2 - 4x)^6[-28x + 5(2 - 4x)]$

$y' = x^4(2 - 4x)^6[-18 - 20x]$

9. $y = \frac{6}{\sqrt{1-2x^3}}$

$y = 6(1-2x^3)^{\frac{1}{2}}$

$y' = 6 \cdot \frac{1}{2}(1-2x^3)^{-\frac{1}{2}} \cdot \frac{d}{dx}(1-2x^3)$

$= 3(1-2x^3)^{-\frac{1}{2}} \cdot -6x^2$

$= -18x^2(1-2x^3)^{-\frac{1}{2}}$

$y' = -\frac{18x^2}{\sqrt{1-2x^3}}$

10. $y = \sqrt[3]{x^2-8}$

$y = (x^2-8)^{\frac{1}{3}}$

$y' = \frac{1}{3}(x^2-8)^{-\frac{2}{3}} \cdot \frac{d}{dx}(x^2-8)$

$= \frac{1}{3}(x^2-8)^{-\frac{2}{3}}.2x$

$y' = \frac{2x}{3\left(\sqrt[3]{x^2-8}\right)^2}$

11. $y = \boldsymbol{sin^2(3x+2)}$

$$y' = 2\sin(3x+2).\frac{d}{dx}\sin(3x+2)$$

$$= 2\ \sin(3x+2).\cos(3x+2).\frac{d}{dx}(3x+2)$$

$$= 2\sin(3x+2).\cos(3x+2).3$$

$$y' = 6\sin(3x+2)\cos(3x+2)$$

12. $y = \frac{x^3}{\sqrt{3x-5}}$

$u = x^3$ $\qquad v = (3x-5)^{\frac{1}{2}}$

$u' = 3x^2$ $\qquad v' = \frac{1}{2}(3x-5)^{-\frac{1}{2}} . \ 3$

$v' = \frac{3}{2\sqrt{(3x-5)}}$

➢ $\frac{vu'-uv'}{v^2}$

$= \frac{(3x-5)^{\frac{1}{2}} .3x^2 - x^3 . \frac{3}{2}(3x-5)^{-\frac{1}{2}}}{\left[(3x-5)^{\frac{1}{2}}\right]^2}$ $\qquad = \frac{x^2\left[9x-15-\frac{3}{2}x\right]}{3x-5}$

$= \frac{x^2(3x-5)^{-\frac{1}{2}}\left[3(3x-5)-\frac{3}{2}x\right]}{(3x-5)^1}$ $\qquad y' = \frac{x^2\left[\frac{15}{2}x-15\right]}{3x-5}$

13. $\boldsymbol{y = x^5 \ln(3x - 2)}$

$u = x^5$ $\qquad v = \ln(3x - 2)$

$u' = 5x^4$ $\qquad v' = \dfrac{1}{(3x-2)} . \dfrac{d}{dx}(3x - 2)$

$v' = \frac{3}{(3x-2)}$

- $\boldsymbol{uv' + vu'}$

$$y' = (x^5)\frac{3}{(3x-2)} + (5x^4)\ln(3x - 2)$$

$$y' = \frac{3x^5}{(3x-2)} + 5x^4 \ln(3x - 2)$$

14. $y = \frac{1}{6(7x+8)^3}$

$$y = \frac{1}{6}(7x+8)^{-3}$$

$$y' = \frac{1}{6} \cdot -3(7x+8)^{-4} \cdot \frac{d}{dx}(7x+8)$$

$$= \frac{1}{6} \cdot -3(7x+8)^{-4} \cdot (7)$$

$$y' = -\frac{7}{2(7x+8)^4}$$

15. $y = \sqrt{3x^3 + 6x}$

$$y = (3x^3 + 6x)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(3x^3 + 6x)^{-\frac{1}{2}} \cdot \frac{d}{dx}(3x^3 + 6x)$$

$$= \frac{1}{2}(3x^3 + 6x)^{-\frac{1}{2}} . (9x^2 + 6)$$

$$= \frac{9x^2+6}{2}(3x^3 + 6x)^{-\frac{1}{2}}$$

$$y' = \frac{9x^2 + 6}{2\sqrt{3x^3 + 6x}}$$

16. $y = \cos(\frac{6-3x}{6+3x})$

$u = 6 - 3x \qquad v = 6 + 3x$
$u' = -3 \qquad v' = 3$

- $\frac{vu' - uv'}{v^2}$

$$= \frac{(6+3x)(-3) - (6-3x)(3)}{(6+3x)^2}$$

$$= \frac{-18 - 9x - [18 - 9x]}{(6+3x)^2}$$

$$y' = -\frac{36}{(6+3x)^2}$$

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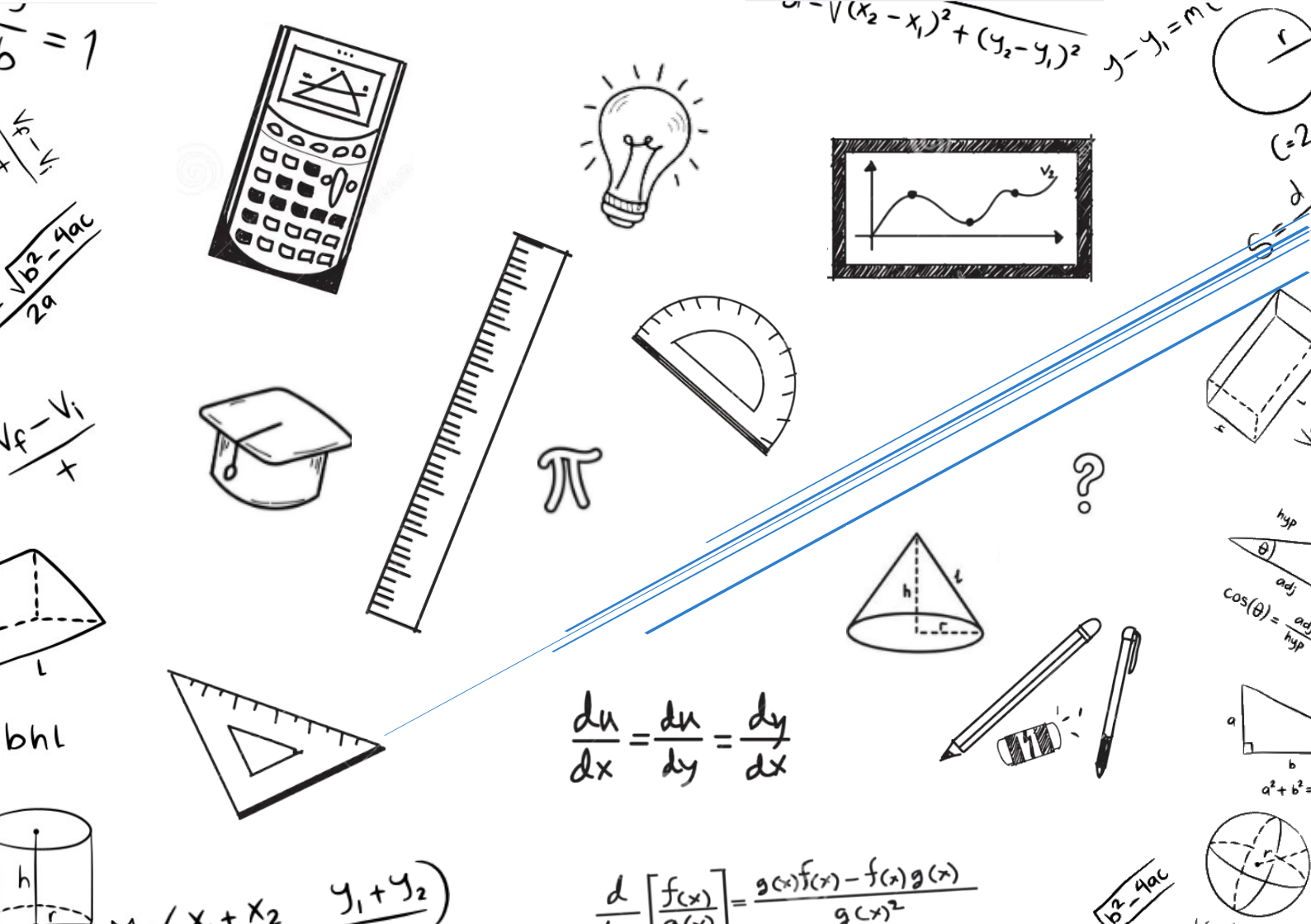
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$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\frac{v_f - v_i}{t}$
bhl
$\cos(\theta) = \frac{adj}{hyp}$
$a^2 + b^2 =$
$\frac{x_1 + x_2}{2}$ $\frac{y_1 + y_2}{2}$