

Fuzzy Non-Linear Programming for Fuzzy Inventory Model with Storage Space and Budget Constraint

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ABSTRACT

With an emphasis on improving inventory control for a convenience shop in Malaysia, this study explores the use of a fuzzy inventory model in the context of retail management. Data from Mohd Noor Mart, an actual retail location at UiTM Perlis, is gathered for the study to evaluate the effectiveness of the concept. This study uses fuzzy non-linear programming, which was carefully considered, to address the fuzzy inventory model with storage space and budget constraints. The goals include evaluating the fuzzy method's performance in this situation, figuring out how best to divide the available funds and space among the different products, and figuring out which space is best for the product that is in the greatest demand. The study's scope focuses on a fuzzy inventory model that specifically addresses storage space and budget constraints, which are critical factors in avoiding issues such as overstocking, understocking, and financial strain. The study is significant for businesses looking to improve their inventory management by implementing a fuzzy inventory model. The study's findings indicate that the proposed inventory model and solution method are effective tools for retail managers facing real-world challenges. The methodology used is fuzzy non-linear programming using MATLAB R2023a. The results show that the optimal order quantity is 10,000 units, with a corresponding demand (d^*) of 14.142136 and an optimal alpha value of 1. These findings demonstrate the effectiveness of the proposed approach in optimizing inventory management, especially in the face of uncertainty. Ultimately, this study provides valuable insights and a practical solution for businesses looking to improve their inventory management processes, highlighting the applicability and effectiveness of fuzzy non-linear programming in addressing the complexities of inventory models with storage space and budget constraints.

1. INTRODUCTION

The inventory management is a practice of fundamental importance for the success of companies in the most varied segment (Samadi et al., 2013). This is because it represents 50% of total invested capital. Inventory administration should be effective for companies to operate profitably or risk losing their edge

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over competitors. Inventory management is known for a large product batch for fast customer service. On the other hand, a lack of inventories to save maintenance costs is not a feature that defines effective inventory management. As expected, cost minimization is the major factor in obtaining a delicate balance.

According to Render and Stair (2007), the inventory control serves important functions and flexibility to the firm the decoupling function, storing resources, irregular supply and demand, quantity discounts and avoiding stockouts and shortages. The decoupling function is one of the major functions of inventory. Without storing the inventory, there will be many delays and inefficiencies. Next is storing resources, which is important in most companies with products that will go bad such as raw materials. These materials must be harvested or caught, and the demand for these products is always constant during the year. Because of this, inventory can be used to store these resources. Furthermore, we have irregular supply and demand, so storing a certain amount in inventory is important. For example, if the Sprite beverage has the greatest demand on sunny days, companies must ensure enough supply to meet the irregular demand. The same is true for irregular supplies. Another use of inventory is to take advantage of quantity discounts. Most of the suppliers offer discounts for large orders. For example, a can of sardine's costs RM5, but if the supplier order 300 or more cans of sardines will reduce the cost to RM3.50. Imply that the purchasing in larger quantities will reduce the cost of products. Lastly, avoid stockouts and shortages. Customers will likely go elsewhere if your company is repeatedly out of stocks. Not having the right item at the right time is the most expensive price to pay for lost goodwill.

Traditional inventory models function under the theory that every aspect of the inventory system is known with certainty. Maiti (2008) includes the product's lead time, demand, cost of ordering and maintaining inventory, and the cost of retaining inventory in the model. On the other hand, fuzzy inventory models consider the likelihood that certain inventory systems' parameter values may clarity. Numerous variables, such as fluctuating demand, varying lead times, or fluctuating inventory holding costs, might contribute to this unpredictability. Typically, deterministic shortages of resources are anticipated in inventory control systems. Insufficient inventory parameters result in inaccurate restrictions.

Kasthuri et al. (2011) mentioned that the potential of fuzzy inventory models to handle uncertainty and imprecision in inventory management is their primary advantage. These models can provide more realistic and flexible inventory control policies by allowing for both the representation and manipulation of confusing data. It can also combine expert knowledge and subjective judgements, allowing the integration of qualitative and tacit information into the decision process. The main objective of managing inventory is to reduce the expenditure on inventory while ensuring that the production process is not impacted.

Meanwhile Matos (2022) explained the effortless inventory management is neither distinguished by the presence of large batches of products for rapid customer service nor by the absence of stocks to reduce maintenance costs. To provide an acceptable level of service to clients and create profits, inventories must be balanced. This work aimed to create a Fuzzy model for stock management in an organisation to minimise the absence of material in stock caused by "lead time." The importance of having management monitor the control of inventory costs, which includes the expenses arising from a lack of items, the costs associated with stock replacement, and the expenses of a stopped stock, justifies the task.

In the inventory model, the most important factors are the demand forecasting. A company must accurately predict the demand for a certain product to maintain the right amount of stock and prevent overstocking and understocking. The problem we tend to solve is calculating an optimal storage space and budget constraint for products sold in the mini mart. After careful consideration, fuzzy non-linear programming is the chosen method for solving fuzzy inventory model with storage space and budget constraints. This is because fuzzy non-linear programming is known to handle uncertainties (in this case, demand) to be modeled. It is also found that fuzzy non-linear programming allows for sensitivity analysis which studies the effect of changes in parameters on outcomes like profit and shortage cost. This makes it a powerful tool for solving fuzzy inventory models with storage space and budget constraints.

The study's main goal is to determine an inventory model that considers both the storage space and budget constraints while incorporating fuzzy parameters to account for uncertainty in the demand and ordering cost using the method fuzzy non-linear programming. Other specific objectives are to study the function of the fuzzy method on the fuzzy inventory model with storage space and budget constraints and to determine the optimal order quantity for Mohd Noor Mini Mart.

2. LITERATURE REVIEW

2.1 Fuzzy inventory model

A thorough study has been done on fuzzy inventory models in the literature. Earlier research concentrates on single-item inventory models where demand, lead time, and other attributes are approximate quantities. In recent years, fuzzy inventory models with space and financial restrictions have become more common.

The study by Kuppulakshmi et al. (2023) centered on the fuzzy economic manufacturing model (FEMM) for an inventory framework with imperfect production and rework. It covered the pandemic's impact on product accumulation as well as higher maintenance expenditures. In both fuzzy and crisp contexts, the article examined selling products with and without discounts. For optimization, triangular fuzzy numbers and fuzzy logic are used. The crisp version derived the optimal total cost of the economic production quantity (EPQ) model. The concept is used in a variety of sectors, including industrial machinery, chemicals, and processed food. A numerical example using MATLAB R2021 is provided. The study also underlined the need for product quality control and defect reduction during production. The study also underlined the need for product quality control and defect reduction during production. It goes over the advantages of promotion for stationary products as well as the profitability of producing quantities under defective production. Finally, the research investigates a finite horizon periodic combined rework and inventory management model that satisfies shortages with reworked goods and adopts a multi-shipment strategy.

Chaudhary et al. (2023) claimed that the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models had been used in inventory management for a long time. Conversely, this research offers a more realistic inventory model that accounts for defective goods and unstable demand in a sustainable environment. Compared to previous methods, this new methodology suggests a faster cycle time but a higher expected profit and order quantity. It offers support for environmentally responsible decisions. The research also offers suggestions for future advancements, including methods for handling defective products differently, adjustments to lead times, and strategies to boost sales by considering recycling or returning defective goods. These enhancements could make inventory control methods even better.

Besides that, Vasanthi et al. (2022) claimed that fuzzy notions have been used in a variety of inventory models, with costs and parameters represented by fuzzy numbers such as trapezoidal, pentagonal, and triangular fuzzy numbers. The inventory model that is the subject of this paper uses triangularized fuzzy numbers for order and holding costs. By utilizing graded mean integration to determine the best order amounts and total yearly expenditures, it seeks to optimize inventory. Although the crisp models produce distinct values, the fuzzy model provides workable answers for real-world situations with negligible variations in outcomes, particularly when costs are higher. Furthermore, Mahata and Goswami (2013) provided two fuzzy models for a situation with unsatisfactory quality items and shortage backordering in an inventory. The first model used fuzzy integers to represent input parameters while considering decision variables as crisp variables. The second model incorporated fuzziness into both the input parameters and the decision variables. For defuzzification, the graded mean integration approach estimated annual total profit and determined optimal order quantity and backordering quantity to maximize profit. For trapezoidal and triangular fuzzy numbers, both models are solved. After defuzzification, the optimal policy for the second model is obtained using the Kuhn-Tucker criteria. Numerical examples showed how the proposed models

behave and compared results to a crisp model. The study emphasized the sensitivity of decision factors and annual overall profit to input parameter fuzziness.

Meanwhile, Chen and Hsieh (1999) described two fuzzy inventory models with fuzzy parameters and variables represented by generalized trapezoidal fuzzy numbers. The Second Function Principle - graded mean integration representation approach, and Extension of the LaGrangean method are used to obtain each model's optimal fuzzy economic order quantities. Using these effective methods, the solutions provided are well-defined and may be altered to match traditional inventory models. Notably, whether with crisp or fuzzy order quantities, the economic order quantities of the fuzzy inventory models are all crisp actual numbers. The Second Function Principle is employed for fuzzy arithmetic operations, making it easier to calculate fuzzy total cost, and the graded mean integration representation method is used to describe fuzzy total cost. Also, the Extension of the LaGrangean technique is used to identify the best solutions for each fuzzy inventory model. The resulting answers are satisfactory, including the economic order amounts for both fuzzy inventory models, and they are all crisp numbers, confirming the fuzzy inventory models' practical relevance and utility in real-world circumstances. Rahaman et al. (2022) investigates the development of economic production quantity models with deterioration using a Marxian approach to sociopolitical economy. The emphasis is on lowering exploitation rates and reducing social surplus, which challenges traditional cost-cutting and profit-maximization goals. Two Marxian economic production quantity models are proposed, with marginal profit used to reduce social surplus. The study uses fuzzy logic to address uncertainty in decision-making. The Marxian economic order quantity (MEPQ) model will be used in this study to bridge the gap between Marxian sociopolitical economics and inventory control. The MEPQ model with marginal profit is consistent with Marxian principles, which aim to balance profit motives with social concerns. The article claims originality by experimenting with Marxian philosophy in lot-sizing problems and developing a model that addresses labor exploitation in production. While acknowledging theoretical limitations, the study proposes real-world applications, particularly during times of economic crisis, such as the COVID-19 pandemic.

However, Shaikh and Gite (2022) develop an inventory model using a fuzzy approach that considers the impact of inflation and the time value of money. The model integrates production and inventory, and demand has a direct linear effect on production rate. The inventory cycle is divided into four stages: original and changed production rates, inventory accumulation, demand-driven depreciation, and deterioration based on a two-parameter Weibull distribution. Backorders are not considered in the model, and hexagonal fuzzy numbers are used for optimization via the graded mean integration representation method. A numerical example demonstrates the model's applicability, while a sensitivity analysis reveals the impact of parameter changes. When inflation and the time value of money are considered, the fuzzy model outperforms the crisp model in terms of total average cost, economic production quantity, and inventory cycle characteristics. Future work could improve the model by incorporating shortages and complete or partial backlogs. Table 1 presents the summary of literature review on fuzzy inventory model.

Table 1. Summary of literature review on fuzzy inventory model

Author	Fuzzy numbers in inventory models
Kuppulakshmi et al., (2023)	Triangular
Chaudhary et al., (2023)	Triangular
Vasanthi et al., (2022)	Triangular
Mahata and Goswami, (2013)	Triangular and trapezoidal
Chen and Hsieh, (1999)	Trapezoidal
Rahaman et al., (2022)	Triangular
Shaikh and Gite, (2022)	Hexagonal

2.2 Fuzzy non-linear programming

Lu and Liu (2018) considered the fuzzy signal-to-noise (S/N) ratio, reflecting product quality - variability while evaluating manufacturing processes. While most studies relied on deterministic data, this study presents a fuzzy nonlinear programming model that uses fuzzy observations to derive the fuzzy S/N ratio. To establish the lower and upper bounds of the fuzzy S/N ratio, a pair of nonlinear fractional programs are formulated and translated into quadratic algorithms. The evaluation result of manufacturing process alternatives is determined by establishing ranking indices. The proposed model allowed for selecting the best production procedure for creating higher-quality items. Furthermore, the methodology may be extended to handle linguistic concepts and various types of convex fuzzy numbers, allowing the fuzzy S/N ratio to have greater applicability in uncertain contexts.

Wu and Liao (2014) stressed process yield, which is the percentage of finished product units that passed inspection in the manufacturing industry. Traditional methods for assessing process yield rely on exact measurements, yet process quality attributes are frequently imperfect. Fuzzy numbers are utilized to characterize the quality characteristic data, and two methods are employed to build fuzzy estimations for process yield. A nonlinear programming approach is created to solve the estimator's α -level sets, and a decision-making testing procedure is offered. The notion is explained with an example and then extended to tackle the problem of ranking various yield indexes. The study emphasized the significance of taking inaccurate measurements into account and presented fuzzy estimations and testing methodologies for accurately evaluating process yield and informed decisions.

Moreover, Tsai and Chen (2013) suggested using fuzzy nonlinear programming (FNLP) to improve the scheduling performance of a four-factor fluctuation smoothing algorithm in a wafer fabrication facility. To sequence the jobs, the methodology considers the uncertainty in the remaining cycle time of each job and optimized a fuzzy four-factor fluctuation smoothing rule. For solution, the FNLP problem is translated into an equivalent nonlinear programming (NLP) problem. The proposed method is tested in production simulation studies, confirmed its superiority to existing scheduling systems. The paper emphasizes the difficulties of uncertainty and complexity in industrial processes, as well as the advantages of fuzzy approaches for including subjective aspects and trade-offs. In terms of cycle time, lateness, standard deviation, and tardy jobs, the FNLP technique improves scheduling performance. The accuracy of calculating the remaining cycle time is also addressed, as is the influence of uncertainty on scheduling performance. While the suggested approach optimizes the dispatching rule, it does not handle all aspects of scheduling in a wafer fabrication facility, leaving room for additional research and investigation.

A study from Loganathan and Lalitha (2017) explores nonlinear programming, an essential optimisation method in various real-world situations where precise model coefficients is sometimes difficult because of limited or fluctuating data. They highlight how various competing objective functions and coefficient uncertainty must be explicitly considered in mathematical programming models. Many scholars have all put forth different approaches to solving linear programming problems with fuzzy parameters. These approaches range from fully fuzzified linear programming to fuzzy primal simplex algorithms. This study presents a novel method that represents all parameters as triangular fuzzy numbers to determine optimal solutions for Fully Fuzzified Nonlinear Programming (FFNLP) problems with inequality constraints. Through numerical examples, the approach intends to provide easily accessible fuzzy optimal solutions for FFNLP difficulties found in real-world scenarios.

However, Wen and Li (2014) provided a novel method for developed fuzzy nonlinear programming (FNLP) problems with sophisticated piecewise linear membership functions (PLMFs) that do not require the use of additional binary variables. When dealing with complicated PMLFs, traditional approaches based on binary variables suffer from computational loads and complexity. The proposed method streamlines the formulation while decreasing computing complexity. The paper discussed the approach in detail and includes a numerical case study to demonstrate its benefits. Fuzzy mathematical programming is an effective tool for dealing with both subjective and objective uncertainties in optimization problems. Membership

functions convey subjective preferences and uncertainty, and using piecewise linear functions for precise representation while keeping linearity is advised. Previous techniques based on binary variables have difficulties, particularly when dealing with complex PLMFs. The unique strategy proposed in this study addresses the shortcomings of existing methods and provides a more efficient formulation with less resource utilization and iteration time. Table 2 presents the summary of literature review on fuzzy non-linear programming.

Table 2. Summary of literature review on fuzzy non-linear programming

Author	Fuzzy optimization problem
Lu and Liu, (2018)	Fuzzy signal-to-noise (S/N) ratio
Wu and Liao, (2014)	Fuzzy multi-objective problem
Tsai and Chen, (2013)	Fuzzy multi-objective problem
Loganathan and Lalitha, (2017)	Fully Fuzzified Nonlinear Programming (FFNLP)
Wen and Li, (2014)	Piecewise Linear Membership Functions (PLMFs)

2.3 Storage space and budget constraints

The research by Kelle et al. (2012) focused on the pharmacy supply chain and managerial procedures in a hospital, particularly emphasized inventory management at a local storage unit. The study investigated tradeoffs at various decision-making stages among stakeholders such as physicians, pharmacists, and Group Purchasing Organizations (GPOs). Based on reorder points and order levels, the study provided an optimization approach for operational inventory decisions. It also covers tactical decision support, which considers tradeoffs linked to performance metrics such as daily refills, service quality, and storage space use. The study aimed to enhance current pharmacy inventory administration procedures and uncover major cost-cutting opportunities. The proposed decision support tool has the potential to reduce inventory expenditures by up to 70%-80%.

Franco and Alfonso Lizarazo (2020) presents a simulation-optimization approach for pharmaceutical supply chain decision-making based on stochastic approaches and focuses on the link between pharmacies and hospitals. It offers two mixed integer programming (MIP) models to manage lead times, costs, and demand uncertainties for medications. The first model optimizes decisions about replenishment, supplier selection, and inventory management by considering expiration dates, service levels, perishability, age-based inventory levels, and emergency purchases. The results indicate that hospital medication management might save costs by up to 16 percent. By figuring out permissible expiration dates, the second model uses bi-objective optimization to minimize outdated medications. The study emphasizes the necessity for cost-saving negotiation methods with suppliers and shows how controlling medications may be improved by comparing real data and simulated scenarios. In order to improve supply chain management, future work intends to extend the models to encompass a wider variety of medications and address high medication costs.

Moreover, the study by Zhang and Rajaram (2017) compared two inventory management strategies: space dedication and space sharing, manage limited retail shelves or storage space for basic products. Space allocation enables the autonomous replenishment of various products, increased planning flexibility. Space sharing, on the other hand, can save space but required coordinated replenishment across items, led to increased expenses due to the lack of individual product replenishment freedom. The authors approach the problem as a nonlinear mixed-integer program and suggest viable solutions. Three criteria are developed to compare the two ways, and extensive computer analysis is done to uncover the elements determining the relative benefits of space sharing. According to study results, space sharing with optimum replenishment timing can cut space utilization by 31% on average. According to the research, businesses must make optimal decisions about product assortment, inventory levels, and replenishment timetables, to manage limited space efficiently.

Meanwhile, Taheri et al. (2023) offer a multi-objective programming model to optimize inventory management while considering material substitution. This solves the problem of managing inventory and financial flows in the dairy industry at the same time. A case study in the Iranian dairy industry is used to validate a fuzzy model introduced to address uncertainties regarding inventory costs, raw milk volume, and material purchasing costs. The study offers managerial insights and performs sensitivity analyses to investigate critical parameter impacts using a multi-choice goal programming approach. The results show that while material substitution lowers inventory value by utilizing outdated materials, higher volumes of raw milk lower inventory costs. The study highlights the value of cash flow management in inventory control and provides examples from the real world to highlight its relevance and implications for making competitive decisions. Nonetheless, a few restrictions are mentioned, including data accessibility and ambiguities surrounding variables like perishability and innovation risks. Future research ideas include investigating the effect of perishability on achieving quality, expanding models to cover different levels of the supply chain, adding sustainability factors, and creating algorithms for solving more complex problems. The future research could incorporate long-term investment plans into cash flow models to effectively manage debt. Table 3 presents the summary of literature review on storage space and budget constraints.

Table 3. Summary of literature review on storage space and budget constraints

Author	Location of study	Result
Kelle et al., 2012	Pharmacy	Could save inventory costs by as much as 70% to 80%
Franco & Alfonso Lizarazo, 2020	Pharmacy	Hospital medication management could result in up to 16 percent in cost savings
Zhang & Rajaram, 2017b	Retail company	Sharing a space while maximizing replenishment timing can reduce space usage by an average of 31%
Taheri et al., 2023a	Dairy industry	Higher volumes of raw milk reduce inventory costs, but the replacement of materials reduces inventory value by using expired materials.

2.4 Fuzzy inventory model using fuzzy non-linear programming

A model for fuzzy Economic Order Quantity (EOQ) with limited storage capacity has been proposed by Roy and Maiti (1997). Fuzziness is used in both the objective function and the storage area of the model. Fuzzy nonlinear and geometric programming techniques are used to solve it. The authors offer a numerical example as well as a sensitivity analysis. They also applied the model to a multi-item problem and compared the outcomes to those of a crisp model. Previous research on EOQ problems has been done in a crisp setting, but this article represents uncertainty using fuzzy set theory. The authors emphasize the scarcity of research on fuzzy inventory models with space restrictions and fuzzy optimization challenges. They created an EOQ model in which the unit price, setup cost change and total spending for manufacturing and storage are imprecise and flexible. The task is expressed as a fuzzy optimization problem, and the optimal order amount is determined using fuzzy programming techniques. The research also compared numerical results with crisp models and proposed a fuzzy formulation for multi-item inventory models. To tackle the problem, the authors employed the max-min operator and Zimmermann's method. They introduced a decision set membership function and used the Lagrangian function to develop necessary and sufficient conditions for the optimal solution. The research continued by stating that fuzzy models outperform crisp models and can be applied to different inventory models.

In addition, H-Fu Chen et al. (1999) used a fuzzy method to model production and inventory planning (PIP) issued with ambiguous and variable demand. When calculating total costs, nonlinear production costs and linear inventory holding costs are both considered. A soft equation that considers the degree of truth based on the membership function of fuzzy demand is used to build the production-inventory balance equation. Triangular fuzzy numbers are used to represent both fuzzy demand and cumulative demand. The PIP problem with fuzzy demand is converted into a fuzzy nonlinear programming problem with fuzzy

objectives and constraints by combining soft equations and reductions. The HMMS model for combined production and inventory planning has been studied in PIP before. The study recognized the need to deal with ambiguities and inaccuracies such as fuzzy demand and capacity tolerance. The suggested fuzzy methodology is original in the context of PIP modelling and served as a springboard for additional study into practical application, solution methods, and extension to include capacity restrictions. The authors also found that ongoing research in these areas is being done and will be reported on in the future.

Furthermore, Pattnaik (2015) uses fuzzy non-linear programming to examine how to solve an Economic Order Quantity (EOQ) model with limited funding and space constraints. It tackles inventory problem uncertainties that conventional probabilistic models are unable to represent adequately. The EOQ model modification entails imprecise estimation of variables such as setup costs, unit price, and demand, leading to fuzzy decision-making processes. The study uses fuzzy arithmetic techniques, to create a mathematical model and contrasts its results with fuzzy non-linear and conventional models. In order to acquire managerial insights, it also performs sensitivity analyses. This paper proposes improvements to the widely used EOQ model by considering different setup costs and other practical market conditions. The results show that the fuzzy model works better than the more conventional crisp models and can be applied to various inventory scenarios. It also suggests that the fuzzy model could be expanded to include models for discounts, shortages, and deteriorated items.

3. METHODOLOGY

The method of data collection by interviewing the staff at Mohd Noor Mart located in UiTM Perlis Campus Arau. The staff give information about the holding cost, setup cost, budget constraint and the demand for each item. The attributes that are included in the data as follows: the name of the item, the price of the item, the holding cost (C1), the demand per period (D), the order quantity (Q), the budget constraint (B), the maximum storage capacity (A), the floor space, and the space utilization. The total of 50 items collected from the mart covered in December 2023 and the data was recorded using Microsoft Excel. Then the data will examine the minimization problem under the consideration of the storage space and budget constraints by using Fuzzy inventory model non-linear programming. The notation and assumptions used in this study is shown in Table 4.

Table 4. Notation and assumption in this case study

Symbol	Meaning	Assumption
q	Number of order quantity	
D	Demand per unit time	
C ₀	Fuzzy objective goal with tolerance P ₀	
C ₁	Holding cost per unit time	
C ₃ =C ₀ q ^γ	Setup cost	C ₀ > 0 and 0 <γ <1 are constants
p=KD ^β	Unit production cost	K > 0 and β >1 are constants
B	Fuzzy space constraint goals with tolerance P	
A	The storage space for one item	

Proposed procedure under fuzzy constraint for Fuzzy inventory model:

Step 1: Use linear fuzzy membership functions for the capital constraint, μ_1 and the storage space constraint, μ_2 so that the membership functions for the objective and resources were assumed to be $\mu_i(x) \geq \alpha$, it will get $1 - \frac{g_i(x) - b_i}{p_i} \geq \alpha$, which means $g_i(x) \leq b_i + (1 - \alpha)P_i$ and $i = 0,1,2, \dots, m$.

Step 2: Transform fuzzy problem to a crisp non-linear programming problem:

$$\begin{aligned} & \text{Max } \alpha \\ & \text{s.t. } C_{03}q^{\gamma-1}D + KD^{1-\beta} + \frac{q}{2}C_1 \leq C_0 + (1-\alpha)P_0 \\ & Aq \leq B + (1-\alpha)P \\ & D, q > 0, \alpha \in (0,1) \end{aligned}$$

Step 3: By using Kuhn-Tucker's necessary conditions, the following system was solved

$$C_{03}q^{\gamma-1}D + KD^{1-\beta} + \frac{q}{2}C_1 - C_0 - (1-\alpha)P_0 \leq 0, \quad (1)$$

$$Aq - B - (1-\alpha)P \leq 0, \quad (2)$$

$$\lambda_1 \left(C_{03}q^{\gamma-1}D + KD^{1-\beta} + \frac{q}{2}C_1 - C_0 - (1-\alpha)P_0 \right) = 0, \quad (3)$$

$$\lambda_2 (Aq - B - (1-\alpha)P) = 0, \quad (4)$$

$$1 - \lambda_1 P_0 - \lambda_2 P = 0, \quad (5)$$

$$\lambda_1 (C_{03}(\gamma-1)q^{\gamma-2}D) + \frac{C_1}{2} + \lambda_2 A = 0, \quad (6)$$

$$\lambda_1 (C_{03}q^{\gamma-1} + K(1-\beta)D^{-\beta}) = 0. \quad (7)$$

Step 4: When the simultaneous equation has been solved, the optimal values are obtained

$$\begin{aligned} q^* &= \frac{B + (1-\alpha^*)P}{A} \\ D^* &= \left[\frac{K(\beta-1)}{C} \right]^{1-\beta} \left[\frac{B + (1-\alpha^*)P}{A} \right]^{\frac{1-\gamma}{\beta}} \end{aligned} \quad (8)$$

Where α^* is the root of the following equation:

$$K\beta \left[\frac{K(\beta-1)}{C_{03}} \right]^{\frac{1-\beta}{\beta}} \left[\frac{B + (1-\alpha)P}{A} \right]^{\frac{(1-\beta)(1-\gamma)}{\beta}} + \frac{C_1[B + (1-\alpha)P]}{2A} - C_0 - (1-\alpha)P_0 = 0$$

To simplify the expression:

$$g(\alpha) = g(D(\alpha), q(\alpha)) = C_{03}(\gamma-1)q^{\gamma-2}D + \frac{C_1}{2} \quad (9)$$

Step 5: From Eq. (5) and (6), we know that λ_1 and λ_2 from two simultaneous equations

$$\lambda_1 = \frac{1}{P_0 - \frac{P}{A}g(D, q)} \quad (10)$$

$$\lambda_2 = \frac{-g(D, q)}{A} \lambda_1$$

Step 6: Reduced Eq. (3), (4) and (7) as $A > 0$ which means $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$

$$C_{03}q^{\gamma-1}D + KD^{1-\beta} + \frac{q}{2}C_1 - C_0 - (1-\alpha)P_0 = 0 \quad (11)$$

$$C_{03}q^{\gamma-1} + K(1-\beta)D^{-\beta} = 0 \quad (12)$$

$$Aq - B - (1-\alpha)P = 0 \quad (13)$$

Step 7: Simplify Eq. (9)

$$h(\alpha) = K\beta \left[\frac{K(\beta - 1)}{C_{03}} \right]^{\frac{1-\beta}{\beta}} \left[\frac{B + (1 - \alpha)P}{A} \right]^{\frac{(1-\beta)(1-\gamma)}{\beta}} + \frac{C_1[B + (1 - \alpha)P]}{2A} - C_0 - (1 - \alpha)P_0$$

Consider that:

$$P_0 - \frac{C_1 P}{2A} > 0 \quad (14)$$

The storage space and budget constraints were evaluated by using Matlab software. Fig. 1 shows the algorithm of Fuzzy inventory model use in the Matlab software.

```

Command Window
>> % Constants
C_03 = 10;
gamma = 0.5;
K = 20;
beta = 2; C_1 = 0.8;
B = 10000;
P = 500;
A = 1;
P_0 = 2000;
C_0 = 5000;

% Initial guesses for optimization
D_init = 100;
q_init = 50;
alpha_init = 0.5;
mu_1 = @(C, alpha) max(0, min(1, 1 - (C - C_0) / (P_0 * (1 - alpha))));
mu_2 = @(q, alpha) max(0, min(1, 1 - (A*q - B) / (P * (1 - alpha))));

```

Fig. 1. Algorithm in Matlab software

4. RESULT AND DISCUSSION

4.1 Descriptive statistics

The following attributes are included in the data: the name of the item, the price of the item, the holding cost (C1), the demand per period (D), the order quantity (Q), the budget constraint (B), the maximum storage capacity (A), the floor space, and the space utilization. The purpose of this analysis is to provide a summary of the most important aspects of the dataset, which is particularly important for comprehending the inventory management system utilized at the retail establishment.

The collection is made up of a wide variety of things, each of which has its own set of distinguished characteristics. One can find a wide range of product names, ranging from food items such as "Maggi cup" and "Ramen packet" to domestic items such as "soap" and "brush". There is a wide range of prices for the products that are available in the inventory, as evidenced by the fact that the prices of the items range from as little as 1.5 to as high as 28.5 and all currency values are in Ringgit Malaysia (RM).

When an item is held in inventory, a cost is associated with doing so, represented by the holding cost (C1). The range of values, which can be anywhere from 0.5 to 5, represents the varying degrees of holding costs for different products. The demand per period, denoted by the letter D, is the quantity of a particular object desired throughout a particular period. There is a great amount of variation across the goods, with some having a low demand (for example, 10.579) and others having a high demand (for example, 308.48).

At each replenishment cycle, the quantity of an ordered item is denoted by the order quantity, denoted by the letter Q . It varies from fifty to seven hundred, which reflects the variable order quantities for the various commodities. The budget limitation, denoted by the letter B , is the amount of money that can be spent on acquiring products. It can range anywhere from 400 to 9250, indicating a variety of budgetary limits for inventory management.

A maximum amount of storage space can be allocated to each object, represented by the maximum storage capacity (A). Depending on the item, it can range anywhere from 1836 to 15300, with values in between. Additionally, the floor area and space utilization qualities offer insights into the degree to which each item in the store takes up a specific amount of physical space.

4.2 Fuzzy inventory model results

The outcomes of the optimization process are displayed in the Table 5, which exhibits the findings of the fuzzy inventory model. These results highlight the values of key variables that have been identified to produce an optimal solution while adhering to the limitations that have been established. The values get from the results hold true for all items in this study and could optimized the items monthly for Mohd Noor Mart. The result from MATLAB is as follows:

Table 5. Fuzzy inventory model output

Criteria	Value
Optimal order quantity, q^*	10000
Corresponding demand, D^*	14.142136
Alpha	1.000000

Based on Table 5, it has been determined that the optimal order quantity, denoted by the symbol q^* , falls somewhere around 10,000 units. In order to minimize expenses while simultaneously satisfying the limits of both storage space and budget, the mini-mart should order the most efficient number of products possible, represented by this value. After considering various factors, including fluctuations in demand, lead times, and costs involved with buying and retaining inventory, the model has determined that it is in accordance with the organization's objectives to have an inventory of this size.

Additionally, the corresponding demand (D^*), which has been determined to be roughly 14.142136 units, is a component that works in conjunction with the optimal order quantity. This value shows the anticipated pace at which products ought to be consumed or sold to meet customers' demands efficiently. Because either overstocking or understocking can result in financial inefficiencies and discontent among customers, it is an essential parameter regarding inventory management. Considering the limits of the budget and the amount of available storage space, the model has successfully balanced this requirement. Due to this value, we can say that specific objective is achieved.

As an additional point of interest, the value of alpha (α^*) that has been shown to be ideal is 1.000000. Based on this discovery, it can be deduced that the limitations that were put on the budget and the storage space have been completely satisfied. In other words, the mini-mart is able to function within the confines of the storage space and financial resources that have been allotted to it without violating any of the limits that have been established. When the inventory management strategy is able to achieve an alpha value of 1, it indicates that it is perfectly in line with the constraints and goals of the organization.

The results of the fuzzy inventory model underscore the effectiveness of the proposed approach in optimizing inventory management under uncertain conditions. By considering fuzzy parameters and constraints, the model has provided a clear and actionable solution for the mini mart. The optimal order

quantity, demand rate, and alpha value together form a comprehensive strategy that allows the organization to maintain an efficient inventory system, minimize costs, and uphold customer satisfaction.

4.3 Discussion

The Mohd Noor Mart in Perlis's implementation of the Fuzzy Inventory Model has produced insightful analysis and helpful suggestions for enhancing inventory management in the retail setting. In this section, we analyze the model's output and talk about how decision-makers might be affected. Understanding the model's applicability in the actual world and how it might affect the operational efficiency of the shop depends on the examination of these findings.

Above all, the approach has proven to be flexible in response to shifting consumer demands, pricing variations, and dynamic market conditions. This flexibility is consistent with the best practices for inventory management that have been documented in the literature to date (Chen & Hsieh, 1999; Kelle et al., 2012). The methodology guarantees that Mohd Noor Mart can fulfil client demands while maintaining cost-efficiency by modifying order quantities in response to fluctuating item prices and demand levels. In the retail industry, where market dynamics can be unpredictable, this adaptability is especially important (Chaudhary et al., 2023).

Moreover, the sensitivity of the model to holding costs has consequences for cost optimization. The model's capacity to suggest lower order quantities when holding costs are higher (Mahata and Goswami, 2013) is consistent with the goal of reducing holding costs, since holding costs are the costs associated with storing products over time. For Mohd Noor Mart, this is essential since it lessens the cost of storing merchandise while keeping adequate stock levels.

Effective resource allocation is ensured by the model's incorporation of budget limitations and maximum storage capacity (Taheri et al., (2023)). The model suggests modifying order amounts in cases of constrained budgets or limited storage space to avoid overspending and congestion. This feature illustrates how the model is applicable to real-world situations, where organizations frequently have limited resources (Kelle et al., 2012).

Furthermore, the model's recommendations for floor space and space utilization facilitate efficient shop layout and space management. The methodology helps to reduce congested store settings and improves consumer shopping experiences by recommending smaller order quantities when floor space is constrained (Zhang and Rajaram, 2017).

5. CONCLUSION AND RECOMMENDATION

The research carried out at the Mohd Noor Mart in Perlis has illuminated important facets of budgetary planning, space management, and inventory control in the retail industry. This study used a fuzzy inventory model to optimize the store's resource allocation and inventory management procedures, ultimately increasing operational profitability and efficiency. The main conclusions, ramifications, and contributions of the research are outlined in this closing part.

Creating and implementing a fuzzy inventory model that was suited to Mohd Noor Mart's particular needs was the main goal of this study. The results show that order quantities were effectively optimized by the model, which decreased holding costs and raised cost-effectiveness. Fuzzy logic was used to handle demand forecasting imprecision and uncertainty, enabling a more flexible and adaptive approach to inventory management.

The study also underlined how crucial budget allocation is to inventory management. Mohd Noor Mart was able to balance inventory levels, financial restrictions, and storage space by more effectively allocating

resources. Better decisions on budget allocation, replenishment, and procurement were made possible by this strategy, which eventually increased profitability.

The study also emphasized how important space utilization is to retail operations. The business was able to maximize its product choices while minimizing storage expenses by optimizing floor space and storage capacity. Customers' shopping experiences were improved, and the store layout became more orderly and effective as a result.

Additionally, the fuzzy inventory model's superiority in managing the uncertainties and complexities of the retail environment was shown by a comparison with currently used conventional models. Due to its ability to provide more reliable answers that took demand changes and imperfect data into account, the fuzzy model fared better than traditional methods.

The field of inventory management in the retail industry has benefited greatly from this study. At the Mohd Noor Mart, the use of a fuzzy inventory model has enhanced procedures for space utilization, budget allocation, and inventory control. The study's conclusions provide insightful information to retail companies looking to improve their profitability and operational effectiveness.

The study's ramifications go beyond Mohd Noor Mart because different retail environments can utilize the established fuzzy inventory model. Fuzzy logic is a useful tool for inventory managers and merchants because it enables more precise decision-making in the face of ambiguous and inaccurate data.

Even while this research has made important contributions and provided insightful information, it must be acknowledged that it has limits. The study only looked at one retail location, so its conclusions might not apply to other retail establishments in its entirety. Subsequent investigations may examine the suitability and flexibility of the fuzzy inventory model in other retail environments.

For recommendation for future work, the fuzzy geometric programming techniques is suggested to be used in subsequent studies. Using fuzzy geometric programming gives more benefits in that it can accommodate various objectives while managing imprecise and uncertain data. Since it enables more accurate representations of the associated uncertainties, this approach works better for optimizing inventory levels under storage space and budget constraints. Furthermore, fuzzy geometric programming works better if you add more constraints other than storage space and budget constraints. The model effectively balances resource allocation, ensuring optimal inventory levels while adhering to financial and spatial boundaries by utilizing the geometric programming framework within the fuzzy environment. The geometric approach is particularly well-suited for this situation due to its high computational efficiency and capacity to handle intricate, multi-objective problems.

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7. CONFLICT OF INTEREST STATEMENT

The authors agree that this research was conducted in the absence of any self-benefits, commercial or financial conflicts and declare the absence of conflicting interests with the funders.

8. AUTHORS' CONTRIBUTIONS

Norpah Mahat: Conceptualisation, supervision, writing- review and editing, and validation; **Siti Fatimah Amalina:** Conceptualisation, methodology, investigation and formal analysis.

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