Forced Response of a High-Static-Low-Dynamic (HSLD) Stiffness Isolator with Active Stiffness Control

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ABSTRACT

The High-Static-Low-Dynamic (HSLD) stiffness vibration isolators have been exploited in many engineering applications due to its capability in having a wider isolation bandwidth, while maintaining the high static load capacities. However, it will lead to a large payload oscillation at the static equilibrium position, if the source of vibration is an oscillating force originating within the payload. In this case, the considerably large resultant motion of the payload will change the system nonlinearity. An active stiffness control for reducing the displacement amplitude of the payload oscillation subjected to a harmonic force excitation is proposed in this paper. The dynamic model of an actively stiffened HSLD stiffness isolator is introduced, and the approximate analytical expression for forced response is obtained using the Harmonic Balance Method (HBM). The obtained forced response curve has demonstrated that the active stiffness control is able to reduce the system's force response, particularly at low frequencies with an approximation of 50%. The nonlinearity of the system becomes smaller as the active stiffness control is applied.

Keywords: *High-Static-Low-Dynamic (HSLD) Stiffness; Vibration Isolator; Active Control*

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Introduction

Vibration control is very important for many engineering structures and machinery components. Excessive vibration can cause structural damage to buildings such as [1], bridges [2], machinery [3]-[4], and other physical structures [5]. The occurrence of damage due to vibration can result in increased maintenance and frequent breakdowns of the equipment, leading to higher costs and productivity losses. In addition, vibrations also can cause discomfort and even health issues for individuals exposed to them. Prolonged exposure to vibrations can lead to fatigue [6]-[7], muscular strain [8], and other physiological problems [9].

Commonly, a passive isolator is applied to attenuate vibrations and isolate sensitive equipment or structures from external disturbances. The isolator performance can be improved by having low stiffness, such that the isolation frequency range increases. However, if the isolator is linear and oriented in the vertical direction, this design will lead to a low load bearing capacity due to the large static displacement.

The High-Static-Low-Dynamic (HSLD) stiffness isolator that possesses a nonlinear force-displacement curve has been proposed in many literatures to overcome the inherent limitation of the linear isolators [10]-[12]. This is due to its capability in obtaining wide isolation bandwidth frequency by lowering the natural frequency of the isolation mount, whilst maintaining the same static load bearing capacity. However, its performance will be affected when the isolated mass on the HSLD stiffness isolator is directly excited by a force which could be an oscillating force originating within the payload itself. This is because the considered HSLD stiffness isolator will produce much softer stiffness at the static equilibrium position when it responds to a dynamic motion. As a result, the isolated mass will experience an excessively large oscillation which is not permitted in practice due to a limited travel space.

In this study, a directly forced mass on a nonlinear HSLD stiffness isolation system with no base excitation is considered. Consistent with previous studies [12]-[16], a Single-Degree-of-Freedom (SDOF) model is adopted in which the HSLD stiffness isolation system is modelled by Duffing's equation with hardening stiffness. The active stiffness control is proposed to stiffen the system such that the large displacement of the isolated mass to an external force or from the force originating within the payload will be reduced.

The organization of this paper is as follows: first, force-displacement characteristic between a HSLD stiffness isolator and its equivalent linear isolator that has exactly the same static stiffness is presented for comparison purposes. Next, the force response and motion transmissibility between a HSLD stiffness isolator and its equivalent linear isolator is demonstrated to highlight the benefits and drawbacks of HSLD stiffness isolator. This is followed by the mathematical modelling of an actively stiffened HSLD stiffness isolation system. The effect of active stiffness on the force response of HSLD stiffness isolation system performance is investigated. Finally, conclusions are given at the end of this paper.

Methodology

Analysis of the force-displacement characteristic between a HSLD stiffness isolator and its equivalent linear isolator

The main feature of the HSLD stiffness isolator is due to its capability in having a low natural frequency for oscillation about its static equilibrium position, and at the same time can withstand static loads without much static displacement. It can thus achieve a wider isolation bandwidth without paying the usual penalty of a high static displacement. This effect is demonstrated in Figure 1, where the force-displacement characteristics of a linear mount (solid line) and a hardening HSLD stiffness mount (dashed line) are compared. Note that, for comparison purposes the static equilibrium position x_o (after the static load has been applied, which is usually the weight of the isolated mass) for both mounts are equivalent. This shows that the static stiffness for linear and hardening HSLD stiffness mount is the same in this case.

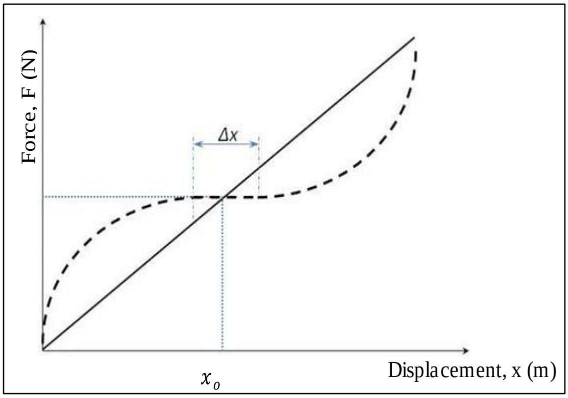

Figure 1: Comparison of force-displacement characteristics between a linear (solid) and a nonlinear spring (dashed), x_o is the equilibrium position for the static load with Δx as the oscillation amplitude. The static stiffness is the same in each case at the equilibrium position

In particular, the stiffness of the HSLD stiffness isolator in this study is assumed to be symmetric about the static equilibrium position, x_o which can be approximated by a cubic function of displacement as; $f = k_1(\Delta x) + k_3(\Delta x)^3$ where Δx is the excursion from the static equilibrium position, ($\Delta x =$

$x - x_o$), k_1 is the coefficient of the linear term, and k_3 is the coefficient of the nonlinear term of cubic restoring force.

Note that, when the system is under small dynamic motion about the static equilibrium position, x_o the dynamic stiffness (local slope of the curve) in the HSLD stiffness mount is less than the equivalent linear mount with a stiffness that gives the same static stiffness. Elsewhere, the dynamic of HSLD stiffness increases which results in the behavior of a hardening system when the excursion from the static equilibrium position is large. In fact, the response of a hardening HSLD stiffness isolator will behave approximately linear if the excursion from the static equilibrium position is small enough. However, when the excursion from the static equilibrium position is large, the system will behave nonlinearly. As a result, for small oscillations, the hardening HSLD stiffness isolator will achieve a lower natural frequency than the linear isolator at the static equilibrium position, x_o.

The natural frequency of the HSLD stiffness system in this work is considered as the linearised natural frequency of the HSLD stiffness system about the static equilibrium position, x_o. In this case, the excursion from the static equilibrium position is considered as infinitesimally small. As a result, the linear term of the restoring force is dominant over the nonlinear term.

Analysis of the force response and motion transmissibility between a HSLD stiffness isolator and its equivalent linear isolator

In order to demonstrate a smaller dynamic stiffness of a HSLD stiffness mount about its static equilibrium position compared to its equivalent linear isolator model; the dynamic stiffness of the HSLD stiffness mount k_1 about its static equilibrium position is stated as:

$$k_1 = \gamma k_{lin} \tag{1}$$

where k_{lin} is the stiffness of the equivalent linear isolator. Note that, the dynamic stiffness of the equivalent linear isolator is greater than the HSLD stiffness isolator, where $0 < \gamma < 1$ as given in Equation (1). By considering small oscillations about the equilibrium position in order to assume linear behaviour, the natural frequency of the linear model with mass, *m* is given by:

$$\omega_l = \sqrt{\frac{k_{lin}}{m}} \tag{2}$$

whilst for the HSLD stiffness mount is:

$$\omega_n = \sqrt{\frac{k_1}{m}} = \sqrt{\gamma}\omega_l \tag{3}$$

It can be noticed that the natural frequency of the linearised HSLD stiffness mount given in Equation (3) is lower than the linear model by a factor of $\sqrt{\gamma}$ as expressed in Equation (2). As a result, there is an extension of the isolation region of the HSLD stiffness mount which is due to the reduction in the natural frequency. Meanwhile, the damping ratio of the equivalent linear system is given as:

$$\zeta_l = \frac{c}{2m\omega_l} \tag{4}$$

whereas the damping ratio of the linearised HSLD stiffness isolator model can be written as:

$$\zeta = \frac{c}{2m\omega_n} = \frac{c}{2m\sqrt{\gamma}\omega_l} = \frac{\zeta_l}{\sqrt{\gamma}} \tag{5}$$

Equation (5) indicates that the reduction in HSLD stiffness mount natural frequency leads to an increment in its damping ratio. Therefore, the HSLD stiffness mount resonance peak will be reduced. For a linear system, the forced response is given by:

$$\frac{X}{F} = \frac{1/k_{lin}}{\sqrt{\left(1-{\Omega_l}^2\right)^2 + 4{\zeta_l}^2{\Omega_l}^2}} \tag{6}$$

where $\Omega_l = \omega/\omega_l$. Therefore, at low frequencies as $\Omega_l \rightarrow 0$, Equation (6) can be simplified as:

$$X = \frac{F}{k_{lin}} = x_o \tag{7}$$

where x_o is the equilibrium position corresponds to the static deflection of the isolator due to the static force, *F*.

Note that as shown in Figure 1, the static deflection for both the linear and HSLD stiffness isolators is the same. Therefore, based on Equation (1), the forced response for a linearised HSLD stiffness model can be expressed as:

$$X = \frac{F}{k_1} = \frac{F}{\gamma k_{lin}} = \frac{x_0}{\gamma} \tag{8}$$

This means that displacement of the payload at the equilibrium position, x_0 for the linearised HSLD stiffness isolator is proportional to x_o/γ. This indicates that the HSLD stiffness isolator will have a larger payload displacement at x_o than the linear model, when the same magnitude of static force is applied. Therefore, the low dynamic stiffness in the HSLD stiffness isolator has the disadvantage of low resistance against a disturbing force at low frequencies. In addition, if the source of vibration is an oscillating force originating within the payload itself and is considerably large, the resultant motion of the payload could be beyond the linearised region. In this case, the forced response curve of the HSLD stiffness isolator will bend at higher frequencies.

The comparison between the transmissibility of the equivalent linear and HSLD stiffness isolator is plotted in Figure 2. The x-axis in the plot is scaled by using $\Omega = \omega/\omega_n$ or $\Omega_l = \sqrt{\gamma}\Omega$ in order to plot the transmissibility curves of HSLD stiffness and linear isolator models on the same graph.

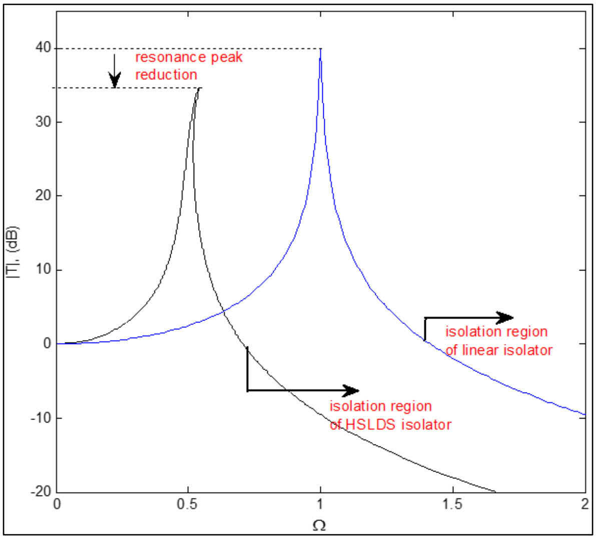

Figure 2: Comparison between absolute transmissibility curves of linear (blue solid line) and HSLD stiffness isolator (black solid line)

Note that, the stiffness of the HSLD stiffness isolator in this plot is based on the linearised stiffness. Therefore, by assuming, $\gamma = 0.25$ the linearised stiffness of the HSLD stiffness isolator is reduced by a factor of four. As a result, the natural frequency of the linearised HSLD stiffness will be halved, which offers better isolation performance than its equivalent linear model. In addition, there is also a reduction in the peak transmissibility of HSLD stiffness isolator. This is because by maintaining constant damping as shown in Equation (5), the damping ratio of the HSLD stiffness isolator will be $\zeta = 0.01$, as the damping ratio of the linear model is $\zeta_l = 0.005$. Therefore, as can be observed in Figure 2 the HSLD stiffness mount offers two main benefits; the first is on the extended isolation region, and the secondly is on the reduction of the resonance peak.

Mathematical modelling of an actively stiffened HSLD stiffness isolation system

A single-degree-of-freedom (SDOF) HSLD stiffness vibration isolator with active stiffness control is shown in Figure 3. The model consists of a mass, m which is mounted on a nonlinear spring (linear stiffness, k_1 and cubic stiffness, k_3), and viscous damper, c in parallel with a secondary force, $f_s(t) = k_{sky}x$ for active stiffness control purposes.

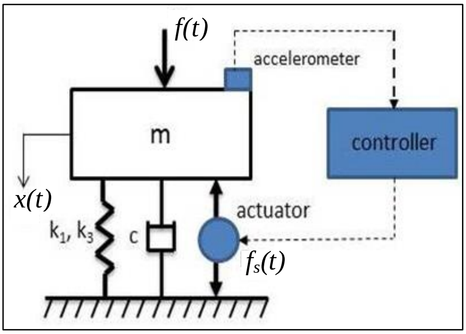

Figure 3: Single-degree-of-freedom (SDOF) hardening Duffing oscillator under active stiffness control subjected to direct harmonic disturbance force

Note that, active stiffness control is based on displacement feedback control, which is proportional to the mass absolute displacement, $x(t)$ where k_{sky} is the control gain from the controller. The system is subjected to a harmonic force excitation, $f(t) = F_P \cos(\omega t)$, where ω is the force excitation frequency. The equation of motion is given by:

$$m\ddot{x} + c\dot{x} + (k_1 + k_{sky})x + kx^3 = F_P \cos(\omega t) \tag{9}$$

In non-dimensional form, this equation becomes:

$$\hat{x}'' + 2\zeta\hat{x}' + (1 + K)\hat{x} + \alpha\hat{x}^3 = \cos(\omega t) \tag{10}$$

where;

$$\zeta = \frac{c}{2m\omega_n}, \qquad {\omega_n}^2 = \frac{k_1}{m}, \qquad \alpha = \frac{k_3{x_o}^2}{k_1}, \qquad \Omega = \frac{\omega}{\omega_n}, \tau = \omega_n t,$$

$$\hat{x}'' = \frac{\ddot{x}}{{\omega_n}^2 x_o}, \quad \hat{x}' = \frac{\dot{x}}{\omega_n x_o}, \quad \hat{x} = \frac{x}{x_o}, \quad K = \frac{k_{sky}}{k_1}$$

and the prime symbol denotes differentiation with respect to non-dimensional time τ. Note that x_o is the displacement of the mass about its static equilibrium position, which is chosen to be based on the linear system, so;

$$x_o = \left.\frac{F_P}{k_1}\right|_{k_3=0,\omega=0} \tag{11}$$

which shows that the static displacement of the mass x_o is determined by the magnitude of applied static force, F_P and stiffness, k_1.

However, the mass will start to oscillate about its static equilibrium position as the oscillating force, F_P varies harmonically (sinusoidal steady-state input force). Note that, the degree of nonlinearity, α in the system will be increased with increasing of x_o. Therefore, it is very important to limit the oscillation of the mass in a good range such that the negative effect on the bending of the resonance curve can be avoided. Since the main concern in this work is to reduce the system's forced response, the proposed active stiffness control has two advantages. The first is in reducing the system's forced response, and the second is in reducing the nonlinearity of the system.

Results and Discussion

The effect of active stiffness control on the force response of the HSLD stiffness isolation system

In this study the Harmonic Balance Method (HBM) to first order expansion is applied with the assumption that the response is to be predominantly at the excitation frequency such that:

$$\hat{x} = \hat{X}\cos(\Omega\tau + \phi) \tag{12}$$

where $\hat{X}$ is the amplitude and ϕ is the phase with respect to the input force.

In this case, the frequency-amplitude relationship is obtained by substituting Equation (12) into Equation (10), to yield:

$$\frac{9}{16}\alpha^2\hat{X}^6 + \frac{3}{2}\alpha(1 + K - \Omega^2)\hat{X}^4 + ((1 + K - \Omega^2)^2 + 4\zeta^2\Omega^2)\hat{X}^2 = 1 \tag{13}$$

which can be written as:

$$\Omega^4 - 2\left[1 + K + \frac{3}{4}\alpha\hat{X}^2 - 2\zeta^2\right]\Omega^2 + \left[\left(1 + K + \frac{3}{4}\alpha\hat{X}^2\right)^2 - \frac{1}{\hat{X}^2}\right] = 0 \qquad (14)$$

Solving Equation (14) for Ω yields:

$$\Omega_1 = \sqrt{\left(1 + K + \frac{3}{4}\alpha\hat{X}^2 - 2\zeta^2\right) - \frac{1}{\hat{X}}\sqrt{1 - 4\zeta^2\hat{X}^2\left(1 + K - \zeta^2 + \frac{3}{4}\alpha\hat{X}^2\right)}} \qquad (15)$$

$$\Omega_2 = \sqrt{\left(1 + K + \frac{3}{4}\alpha\hat{X}^2 - 2\zeta^2\right) + \frac{1}{\hat{X}}\sqrt{1 - 4\zeta^2\hat{X}^2\left(1 + K - \zeta^2 + \frac{3}{4}\alpha\hat{X}^2\right)}} \qquad (16)$$

Accordingly, based on Equations (15) and (16), the effect of active stiffness on the forced response of a Duffing oscillator is plotted in Figure 4. For comparison purposes the equivalent linear model with stiffness k_1 (blue solid line) is plotted on the same graph.

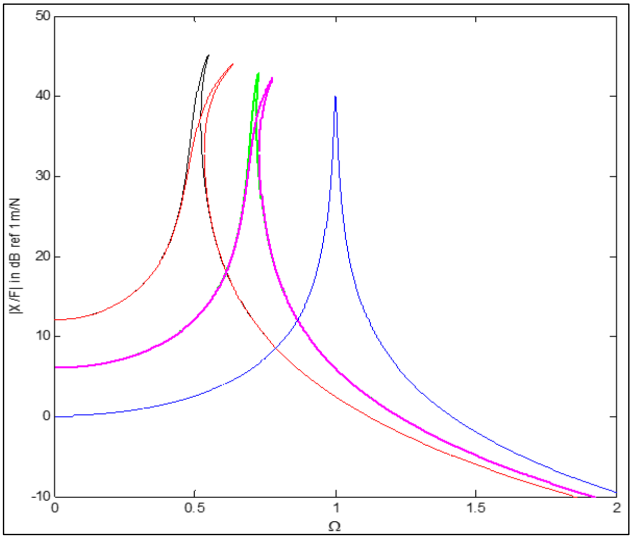

Figure 4: The effect of active stiffness control on the force response of Duffing oscillator with two different nonlinearities. Black line ($\alpha = 1.3x10^{-4}$), red line ($\alpha = 3.3x10^{-4}$), green line ($\alpha = 1.3x10^{-4}, K = 1$), magenta line ($\alpha = 3.3x10^{-4}, K = 1$) and blue solid line is equivalent linear model

It is worth mentioning that the Duffing oscillator is chosen to have a lower dynamic stiffness (k_1 = 0.25 N/m) than the equivalent linear model (k_1 = 1 N/m), which produces a higher forced response. Note that the black line represents a weak nonlinearity of the Duffing oscillator with α = 1.3x10^(-4), while the red solid line denotes the strong nonlinearity of the Duffing oscillator with α = 3.3x10^(-4) (due to larger amplitude of mass oscillation x_0). It can be observed that, when active stiffness is applied (K = 1), the forced response for both systems is reduced, particularly at low frequencies with an approximation of 50%. This is demonstrated in the green and magenta solid line for systems of α = 1.3x10^(-4) and α = 3.3x10^(-4) respectively. In fact, as expected the resonance is shifted to a higher frequency, and the bending of the forced response curve is reduced as active stiffness is applied. This indicates that the system has become less nonlinear, and the resonance frequency can be returned to Ω = 1 with an appropriate active stiffness value. In this case, the forced response is equivalent to the linear model but the active control has removed the benefit of the HSLD stiffness isolator in having a low natural frequency system.

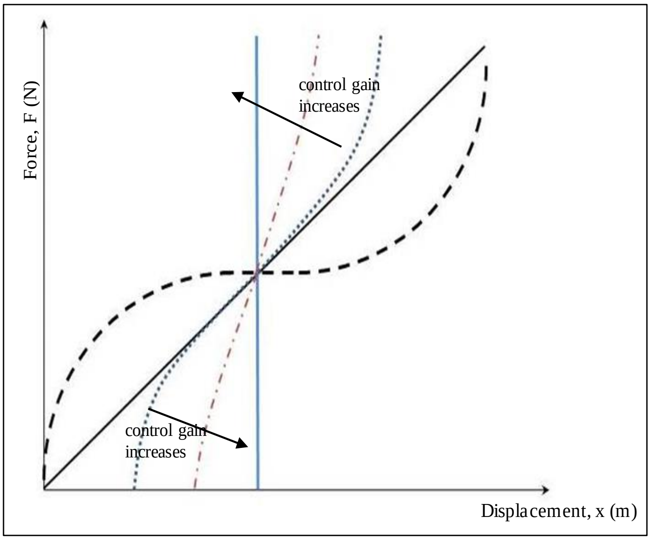

Figure 5: The effect of active stiffness control on the force–displacement curve of a Duffing oscillator. Actively stiffened Duffing oscillator denoted by dotted, and dash-dotted line, infinite active stiffness control gain (blue solid line), passive Duffing oscillator (dashed solid line), and equivalent linear model (black line), where x_o is the static equilibrium position.

The influence of active stiffness control on the forced response of a Duffing oscillator can be described in the force-displacement relation as illustrated in Figure 5. First, it is apparent that the equilibrium position, x_o does not change with the application of active stiffness on the system. The equivalent linear model (black solid line) that has the same static stiffness is shown for comparison. As the control gain increases, the system becomes stiffer to result in a less nonlinear response. In addition, the system's dynamic stiffness also increases which is shown by the increment on the slope of the force-displacement curve. This finding leads to an agreement with the previous study that highlights the role of active stiffness in stability improvement [17]-[21]. Apart from that, when the control gain is infinite, the mass will be totally isolated and stay motionless at the static equilibrium position. This situation is denoted by the blue vertical line in the plot, which corresponds to a perfect isolation performance.

There are two major limitations in this study that could be addressed in future research. First, the study only considered harmonic force excitation. Future work could explore the effect of random force excitation on the HSLD stiffness isolation system. Second, the result of the study focused on numerical simulation which is based on the derived mathematical modelling. Therefore, an experimental work is required for validation in the future work.

Conclusion

In this paper, the effect of active stiffness control on the forced response of the HSLD stiffness isolator subjected to a harmonic force disturbance is presented. The obtained forced response curve has demonstrated that the active stiffness control is able to reduce the system's force response, particularly at low frequencies with an approximation of 50%. As a result, the oscillation of the payload subjected to harmonic direct disturbance force is lesser with the increment of control gain. Consequently, the nonlinearity of the system becomes smaller as the active stiffness control is applied. In fact, the active stiffness control only changes the dynamic stiffness but not the static equilibrium position of the system. The description of the influence of active stiffness on a Duffing oscillator is presented on the plot of force-displacement relation.

Contributions of Authors

The authors confirm the equal contribution in each part of this work. All authors reviewed and approved the final version of this work.

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Conflict of Interests

All authors declare that they have no conflicts of interest.

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