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JACCARD INDEX WITH YAGER CLASS T-NORM FOR RANKING FUZZY NUMBERS

Nazirah Ramli, Nurul Haziqah Mohamad, Marina Mohamad and Fairuz Shohaimay
Universiti Teknologi MARA Pahang, Malaysia

ABSTRACT

This study proposes a new method for ranking fuzzy numbers based on Jaccard similarity measure index and Yager class t-norm. The procedure of this new ranking method involves six phases which are determining fuzzy maximum and fuzzy minimum, intersection and union of fuzzy numbers, scalar cardinality of fuzzy numbers, fuzzy evidence, fuzzy total evidence and pairwise ranking. The result shows that for certain conditions, the ranking is affected by the values of w and has improved some of the previous results that cannot discriminate the ranking of the fuzzy numbers. Results from this study can be of practical significance to fuzzy decision-making in real situations.
Keywords: Ranking fuzzy numbers, Jaccard index, Yager class t-norm, decision-making

INTRODUCTION

In fuzzy environment, the ranking of fuzzy numbers is a prerequisite procedure for decision-making problems. Fuzzy numbers are employed to describe the performance of alternatives and the selection of alternatives will eventually lead to the ranking of corresponding fuzzy numbers. However, ranking of fuzzy numbers is not an easy task since fuzzy numbers are represented by possibility distributions and they can overlap with each other. Various methods of ranking fuzzy numbers (RFNs) have been developed but no method can rank fuzzy numbers satisfactorily in all cases and situations. Some methods produce non-discriminate and non-intuitive results, limited to normal and triangular types of fuzzy numbers and only consider neutral decision makers' perspective. There are also methods that produce different ranking results for the same situations and some have the difficulty of interpretation.

In 1998, Cheng proposed a distance index based on the centroid concept and CV index for RFNs. However, in some situations, the ranking result by the distance index contradicts with the result by the CV index. Thus, to overcome the problems, Chu and Tsao (2002) proposed an area between the centroid point and original point as the ranking index. Chen and Chen (2007) then, found that Cheng's (1998) distance index and Chu and Tsao's (2002) methods cannot rank correctly two fuzzy numbers having the same mode and symmetric spread. Furthermore, Asady and Zendehnam (2007) introduced distance minimization concept for RFNs but their method cannot discriminate the ranking of embedded fuzzy numbers (Hajjari and Barkhordary, 2007). In other studies by Wang et al. (2009), they proposed a ranking method based on deviation degree of the fuzzy numbers. However, the method cannot rank fuzzy numbers and images consistently and thus, Asady (2010) suggests a correction on the left and right deviation degree used in Wang et al.'s (2009). Furthermore, Hajjari and Abbasbandy (2011) have pointed out that Asady's (2010) also has shortcoming in which the method does not able to rank fuzzy numbers in all situations correctly. Recently, Ramli and Mohamad (2012) proposed a method based on Ochiai index and Hurwicz criterion for RFNs. This method can successfully discriminate the ranking of two fuzzy numbers having the same mode and symmetric spread that fails to be discriminated by many researchers. However, the method also has limitation such that it cannot discriminate the ranking of some fuzzy numbers having the same values of area. Consequently, the quest of finding a method for RFNs is still a current issue since all the aforementioned methods have shown shortcomings.

The main aim of this study is to propose a ranking method based on Jaccard index with Yager class t-norm. Jaccard is a set theoretic type of similarity measure index which is commonly used in pattern recognition, and t-norm is a binary algebraic operation on the unit interval. The ranking behaviour of the proposed ranking method is investigated. The paper is organized as follows. Section 2 briefly reviews the preliminary concepts of fuzzy numbers. In Section 3, the Jaccard index with Yager class t-norm for RFNs is proposed. Section 4, presents six numerical examples to illustrate the advantages of the proposed method. Lastly, the paper is concluded in Section 5.

PRELIMINARIES

In this section, basic concept on fuzzy numbers is reviewed from Dubois and Prade (1978).

A fuzzy number is a fuzzy set in the universe of discourse X with the membership function defined as,

$$\mu_A(x)=\begin{cases}\mu_A^L(x) & ,a\le x\le b\\ w & ,b\le x\le c\\ \mu_A^R(x) & ,c\le x\le d\\ 0 & ,\text{otherwise}\end{cases}$$

where $\mu_A^L:[a,b]\to[0,w]$, $\mu_A^R:[c,d]\to[0,w]$, $w\in(0,1]$, μ_A^L and μ_A^R denote the left and the right membership functions of the fuzzy number A .

If the membership function $\mu_A(x)$ is a piecewise linear, then A is called as a trapezoidal fuzzy number with membership function defined as

$$\mu_A(x)=\begin{cases}w\left(\dfrac{x-a}{b-a}\right) & ,a\le x\le b\\ w & ,b\le x\le c\\ w\left(\dfrac{d-x}{d-c}\right) & ,c\le x\le d\\ 0 & ,\text{otherwise}\end{cases}$$

and denoted as $A=(a,b,c,d;w)$. If $b=c$, then the trapezoidal becomes a triangular fuzzy number denoted as $A=(a,b,d;w)$.

PROPOSED JACCARD RANKING INDEX WITH YAGER CLASS T-NORM

Based on the psychological ratio model of similarity from Tversky (1977) which is defined as,

$$S_{\alpha,\beta}(X,Y)=\frac{f(X\cap Y)}{f(X\cap Y)+\alpha.f(X-Y)+\beta.f(Y-X)}$$

various index of similarity measures have been proposed. For $\alpha=1$ and $\beta=1$, the ratio model of similarity becomes the Jaccard similarity measure index which is defined as,

$$S_{1,1}(X,\dot{Y})=\frac{2.f(X\cap Y)}{f(X)+f(Y)}$$

Typically, the function f is taken to be the cardinality function. The objects X and Y described by the features are replaced with fuzzy numbers A and B which are described by the membership functions. The fuzzy Jaccard is defined as,

$$S_J(A,B)=\frac{|A\cap B|}{|A\cup B|},$$

where $|A|$ denotes the cardinality of A , $\cap$ and $\cup$ are the t-norm and s-norm respectively. The procedure for fuzzy Jaccard ranking index with Yager class t-norm is as follows:

Step 1: For each pair of the fuzzy numbers A_i and A_j, find the fuzzy maximum and fuzzy minimum of A_i and A_j.

Step 2: Find $A_i \cap MAX(A_i, A_j)$, $A_i \cup MAX(A_i, A_j)$, $A_j \cap MIN(A_i, A_j)$, $A_j \cap MIN(A_i, A_j)$, $A_j \cap MAX(A_i, A_j)$, $A_j \cup MAX(A_i, A_j)$, $A_i \cap MIN(A_i, A_j)$, $A_i \cup MIN(A_i, A_j)$.

Step 3: Calculate $|A_i \cap MAX(A_i, A_j)|$, $|A_i \cup MAX(A_i, A_j)|$, $|A_j \cap MIN(A_i, A_j)|$, $|A_j \cap MIN(A_i, A_j)|$, $|A_j \cap MAX(A_i, A_j)|$, $|A_j \cup MAX(A_i, A_j)|$, $|A_i \cap MIN(A_i, A_j)|$, $|A_i \cup MIN(A_i, A_j)|$.

Step 4: Calculate the evidences of $E(A_i \succ A_j)$, $E(A_j \prec A_i)$, $E(A_j \succ A_i)$ and $E(A_i \prec A_j)$ which are defined based on Jaccard index as, $E(A_i \succ A_j) = S_J(MAX(A_i, A_j), A_i)$, $E(A_j \prec A_i) = S_J(MIN(A_i, A_j), A_j)$, $E(A_j \succ A_i) = S_J(MAX(A_i, A_j), A_j)$ and $E(A_i \prec A_j) = S_J(MIN(A_i, A_j), A_i)$. To simplify, C_{ij} and c_{ji} are used to represent $E(A_i \succ A_j)$ and $E(A_j \prec A_i)$, respectively. Likewise, C_{ji} and c_{ij} are used to denote $E(A_j \succ A_i)$ and $E(A_i \prec A_j)$ respectively.

Step 5: Calculate the total evidences $E_{total}(A_i \succ A_j)$ and $E_{total}(A_j \succ A_i)$ which are defined based on the aggregation of evidences with Yager class t-norm as $E_{total}(A_i \succ A_j) = 1 - \min\left[1, \left[(1 - C_{ij})^w + (1 - c_{ji})^w\right]^{\frac{1}{w}}\right]$ and $E_{total}(A_j \succ A_i) = 1 - \min\left[1, \left[(1 - C_{ji})^w + (1 - c_{ij})^w\right]^{\frac{1}{w}}\right]$ such that $w \in (0, +\infty)$. To simplify, $E_J(A_i, A_j)$ and $E_J(A_j, A_i)$ are used to represent $E_{total}(A_i \succ A_j)$ and $E_{total}(A_j \succ A_i)$, respectively.

Step 6: For each pair of the fuzzy numbers, compare the total evidences in Step 5 which will result the ranking of the two fuzzy numbers A_i and A_j as follows:

i. $A_i \succ A_j$ if and only if $E_J(A_i, A_j) > E_J(A_j, A_i)$.

ii. $A_i \prec A_j$ if and only if $E_J(A_i, A_j) < E_J(A_j, A_i)$.

iii. $A_i \approx A_j$ if and only if $E_J(A_i, A_j) = E_J(A_j, A_i)$.

IMPLEMENTATION

In this section, six sets of numerical examples are presented to illustrate the validity and advantages of fuzzy Jaccard ranking index with Yager class t-norm. Table 1 shows the ranking results.

Set 1: $A_1 = (0,1,2)$, $A_2 = \left(\frac{1}{5}, 1, \frac{7}{4}\right)$, Set 2: $A_1 = (3,6,9)$, $A_2 = (5,6,7)$,

Set 3: $A_1 = (3,6,9)$, $A_2 = (5,8,11)$, Set 4: $A_1 = \left(\frac{94}{35}, \frac{46}{7}, 10\right)$, $A_2 = (2,7,9)$,

Set 5: $A_1 = (3,6,11,14)$, $A_2 = (2,7,10,15)$, Set 6: $A_1 = (-1,0,1)$, $A_2 = (-4,1,2)$.

Table 1: Comparative ranking result of fuzzy Jaccard index with Yager class t-norm

Fuzzy Numbers	Proposed Ranking Index: Evidences, Total Evidences	Proposed Ranking Index: Ranking Result	Ranking Result of Previous Studies
Set 1	$C_{12}=0.9$, $c_{21}=0.872$, $C_{21}=0.7569$, $c_{12}=0.7813$, $E_J(A_1,A_2)=1-\min\left[1,\left[(0.1)^w+(0.128)^w\right]^{\frac{1}{w}}\right]$, $E_J(A_2,A_1)=1-\min\left[1,\left[(0.2431)^w+(0.2187)^w\right]^{\frac{1}{w}}\right]$	$A_1\approx A_2$, $w\in(0,0.318]$ $A_1\succ A_2$, $w\in[0.319,1000]$	Cheng (1998) and Liang et al. (2006): $A_1\prec A_2$ Chu and Tsao (2002) and Deng et al. (2006): $A_1\succ A_2$ Ramli (2012): $A_1\succ A_2$, $\beta\in[0,1]$
Set 2	$C_{12}=0.667$, $c_{21}=0.5$, $C_{21}=0.5$, $c_{12}=0.667$, $E_J(A_1,A_2)=1-\min\left[1,\left[(0.3333)^w+(0.5)^w\right]^{\frac{1}{w}}\right]$ $E_J(A_2,A_1)=1-\min\left[1,\left[(0.3333)^w+(0.5)^w\right]^{\frac{1}{w}}\right]$	$A_1\approx A_2$, $w\in(0,+\infty)$	Cheng (1998), Chu and Tsao (2002), Asady and Zendehnam (2007), Wang and Lee (2008): $A_1\approx A_2$ Wang et al. (2005): $A_1\succ A_2$ Wang et al. (2009): $A_1\prec A_2$
Set 3	$C_{12}=0.2857$, $c_{21}=0.2857$, $C_{21}=1$, $c_{12}=1$, $E_J(A_1,A_2)=1-\min\left[1,\left[(0.7143)^w+(0.7143)^w\right]^{\frac{1}{w}}\right]$, $E_J(A_2,A_1)=1$	$A_1\prec A_2$, $w\in(0,1000]$	Chen and Lu (2002): $A_1\prec A_2$, $\beta=0,0.5,1$ Ramli (2012): $A_1\succ A_2$, $\beta\in[0,1]$
Set 4	$C_{12}=0.872$, $c_{21}=0.958$, $C_{21}=0.8967$, $c_{12}=0.8550$, $E_J(A_1,A_2)=1-\min\left[1,\left[(0.128)^w+(0.042)^w\right]^{\frac{1}{w}}\right]$, $E_J(A_2,A_1)=1-\min\left[1,\left[(0.1033)^w+(0.145)^w\right]^{\frac{1}{w}}\right]$	$A_1\succ A_2$, $w\in(0,1000]$	Chen and Lu (2002): $A_1\succ A_2$, $\beta=0,0.5$ $A_1\approx A_2$, $\beta=1$ Lu and Wang (2005): $A_1\prec A_2$ Ramli (2012): $A_1\succ A_2$, $\beta\in[0,1]$
Set 5	$C_{12}=0.667$, $c_{21}=0.8338$, $C_{21}=0.4991$, $c_{12}=0.3992$, $E_J(A_1,A_2)=1-\min\left[1,\left[(0.3332)^w+(0.1661)^w\right]^{\frac{1}{w}}\right]$, $E_J(A_2,A_1)=1-\min\left[1,\left[(0.5008)^w+(0.6007)^w\right]^{\frac{1}{w}}\right]$	$A_1\succ A_2$, $w\in(0,1000]$	Wang and Luo (2009): $A_1\succ A_2$, $\beta=0$ $A_1\approx A_2$, $\beta=0.5$ $A_1\prec A_2$, $\beta=1$ Ramli (2012): $A_1\approx A_2$, $\beta\in[0,1]$
Set 6	$C_{12}=0.5$, $c_{21}=0.6944$, $C_{21}=0.276$, $c_{12}=0.451$, $E_J(A_1,A_2)=1-\min\left[1,\left[(0.5)^w+(0.3056)^w\right]^{\frac{1}{w}}\right]$, $E_J(A_2,A_1)=1-\min\left[1,\left[(0.7224)^w+(0.549)^w\right]^{\frac{1}{w}}\right]$	$A_1\approx A_2$, $w\in(0,0.756]$ $A_1\succ A_2$, $w\in[0.757,1000]$	Wang et al. (2009): $A_1\prec A_2$ Asady and Zendehnam (2007): $A_1\approx A_2$

The ranking results of some fuzzy numbers (Sets 1 and 6) are affected by the values of w. For Set 2, the values of total evidences are the same, thus produces the equal ranking results for $w\in(0,+\infty)$. The ranking results of other fuzzy numbers are not affected by the values of w.

CONCLUSION

This paper presents a ranking method using Jaccard index with Yager class t-norm. The ranking results of some fuzzy numbers are affected by the values of *w*. The ranking results have also improved some of the fuzzy numbers in Set 5 (which have the same values of area) and Set 6, which cannot be discriminated by Ramli (2012) and Asady and Zendehnam (2007) respectively. However, the ranking method cannot discriminate the fuzzy numbers in Set 2 which have the same mode and symmetric spread. In many fuzzy decision-making problems, the output obtained is normally in the types of overlapped fuzzy numbers as in Set 3 which has the property of $MAX(A_1, A_2) = A_2$ and $MIN(A_1, A_2) = A_1$. By this ranking method the decision makers preferred to choose A_2 compared to A_1 which is consistent with human intuition. This ranking method not only provide the conclusion of preferred or not preferred of the alternatives, but can also represent the imprecise relation between alternatives according to the degree of preference such as A_2 dominates A_1, A_2 is slightly better than A_1 or A_2 and A_1 are more or less the same, and others. Thus, this new ranking method is a valuable tool for fuzzy decision-making as it can provide detail information on the degree of preference of decision makers.

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