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Modal Extraction Accuracy Using Single Station Time Domain (SSTD) Technique

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ABSTRACT

Single Station Time Domain (SSTD) technique is one of the efficient methods used to extract modal parameters. This technique relies on free decay responses to obtain the required parameters. In this paper, a study on time shift interval on SSTD algorithm method is investigated. It is found that altering specific time interval settings could improve the accuracy of natural frequencies and damping ratios determination of a structure. Accuracy assessment of the method is made using simulated analytical data with known properties where percentage error between the results of simulated and this method can be calculated and compared.

Keywords: *Ibrahim Time Domain (ITD); Single Station Time Domain (SSTD); Frequency response function (FRF), Damping ratio*

Introduction

Various modal extraction methods have been used to extract the dynamic properties of a vibrating system in the forms of natural frequencies, mode shapes and damping ratios. In the past four decades, numerous techniques had been developed in order to obtain more reliable identification of these parameters. Generally, these techniques can be divided into two categories i.e. time and frequency domain methods.

Methods in time domain process the data measured from a structure that undergoes vibration in terms of time signal. The time signal used is either original

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or a constructed new time signal mainly from the Frequency Response Function (FRF) of a structure known as Impulse Response Function (IRF). Ibrahim Time Domain (ITD) [1] and Random Decrement Method (RD) [2] are methods that utilize original time signal while Polyreference Complex Exponential Method (PRCE) [3] is one example that uses Impulse Response Function (IRF).

In frequency domain, the original time signal is transformed to FRF using FFT [4]. Examples of these methods are Rational Fraction Polynomial [5], Frequency Domain Prony [6] and Complex Exponential Frequency Domain [7]. However, working in frequency domain leads to leakage problem and thus, explicitly highlights the advantage of time domain [8].

The purpose of this paper is to study Single Station Time Domain *(*SSTD) method and observe the accuracy of modal parameters obtained with respect to a number of different time shift interval introduced in the algorithm. The method is modified so that it can work as Single Input Multiple Output (SIMO) type instead of original Single Input Single Output (SISO) type in order to produce more accurate results.

Theoretical Background

In this section, the algorithm for SSTD as described in [9] is outlined. Parts of the algorithm where different time shift interval needed to be experimented to observe the accuracy of natural frequency and damping ratio will also be explained.

Single Station Time Domain (SSTD) Algorithm

SSTD method has similar algorithm structure with another technique known as Ibrahim Time Domain (ITD) method. The difference between these approaches is ITD is a SIMO type while SSTD is a SISO type technique. SSTD algorithm utilizes the time response at unique location in order to find modal parameters. Considering at time t_j, a system with *N* structural modes can be expressed as a summation of the individual responses specific at point *i* and time *j* of each mode *r*:

$$x_i(t_j) = \sum_{r=1}^{2N} p_{ir} e^{s_r t_j} \tag{1}$$

where p_{ir} is the eigenvector at point *i* for mode *r*. Although initially *N* refers to the number of structural modes, in this algorithm, the prefix is specifically pointing to the number of identification order which will identify converged modal parameters value. The total number of converged modal parameters is referred to as the number of structural mode for that system. From (1), the response measured at *L* instances of time can be written in the following matrix form:

$$\underset{2N\times L}{[\boldsymbol{X}]} = \underset{2N\times 2N}{[\boldsymbol{P}]}\,\underset{2N\times L}{[\ddot{\mathbf{E}}]} \tag{2}$$

Another set of equation can be written in matrices which are exactly similar to (2) but is shifted by Δt_1 as written below:

$$\begin{aligned} x_i(t_j + \Delta t_1) &= \sum_{r=1}^{2N} p_{ir}\, e^{s_r(t_j + \Delta t_1)} \\ &= \sum_{r=1}^{2N} \hat{p}_{ir}\, e^{s_r t_j} \end{aligned} \tag{3}$$

Where,

$$\hat{p}_{ir} = p_{ir}\, e^{s_r \Delta t_1} \tag{4}$$

In symbolic form, (3) can be expressed as below:

$$\underset{2N\times L}{[\hat{\boldsymbol{X}}]} = \underset{2N\times 2N}{[\hat{\boldsymbol{P}}]}\,\underset{2N\times L}{[\ddot{\mathbf{E}}]} \tag{5}$$

A square matrix $[A_s]$ of order $2N$ is defined as below:

$$\underset{2N\times 2N}{[\hat{\boldsymbol{P}}]} = \underset{2N\times 2N}{[\boldsymbol{A_s}]}\,\underset{2N\times 2N}{[\boldsymbol{P}]} \tag{10}$$

Multiply (3) by $[A_s]$ gives

$$\underset{2N\times 2N}{[\boldsymbol{A_s}]}\,\underset{2N\times L}{[\boldsymbol{X}]} = \underset{2N\times 2N}{[\boldsymbol{A_s}]}\,\underset{2N\times 2N}{[\boldsymbol{P}]}\,\underset{2N\times L}{[\ddot{\mathbf{E}}]} \tag{11}$$

Insert (10) into (11) gives

$$\underset{2N\times 2N}{[\boldsymbol{A_s}]}\,\underset{2N\times L}{[\boldsymbol{X}]} = \underset{2N\times 2N}{[\hat{\boldsymbol{P}}]}\,\underset{2N\times L}{[\ddot{\mathbf{E}}]} \tag{12}$$

Substitute (5) into (12) gives

$$\underset{2N\times 2N}{[\boldsymbol{A_s}]}\,\underset{2N\times L}{[\boldsymbol{X}]} = \underset{2N\times L}{[\hat{\boldsymbol{X}}]} \tag{13}$$

However, (13) utilizes a single output signal with respect to a single input. In order to obtain a SIMO variant of SSTD, a least square approach is used as Matrix $[A_s]$ is independent of the location of measurements and remain valid for any SISO combination [1]. Hence, (13) for *n* responses due to single input can be expressed as below:

$$\underset{2N\times 2N}{[\boldsymbol{A_s}]} \underbrace{\begin{bmatrix} \underset{2N\times L}{\boldsymbol{X_1}} & \underset{2N\times L}{\boldsymbol{X_2}} & \cdots & \underset{2N\times L}{\boldsymbol{X_n}} \end{bmatrix}}_{2N\times nL} = \underbrace{\begin{bmatrix} \underset{2N\times L}{\hat{\boldsymbol{X}}_1} & \underset{2N\times L}{\hat{\boldsymbol{X}}_2} & \cdots & \underset{2N\times L}{\hat{\boldsymbol{X}}_n} \end{bmatrix}}_{2N\times nL} \tag{14}$$

where $[\boldsymbol{X_n}]$ refer to matrix with point *i* and time *j* as explained in (1). In more compact form,

$$\underset{2N\times 2N}{[\boldsymbol{A_s}]}\,\underset{2N\times nL}{[\boldsymbol{Y}]} = \underset{2N\times nL}{[\hat{\boldsymbol{Y}}]} \tag{15}$$

Matrix [A_s] can be found by using pseudo inverse technique to give an expression known as Double Least-Squares (DLS). It is claimed that the equation below yield better estimates of the damping factors [15].

$$[A_S] = \frac{1}{2}\ ([\hat{Y}][\hat{Y}]^T)([Y][\hat{Y}]^T)^{-1} + ([\hat{Y}][Y]^T)([Y][Y]^T)^{-1} \tag{16}$$

The eigenvalues are deduced from matrix [A_s] as stated by (10) can also be written as follow:

$$\underset{2N\times 2N}{[\boldsymbol{A_s}]}\underset{2N\times 1}{\begin{Bmatrix} p_r e^{s_r 0\Delta t_1} \\ p_r e^{s_r 1\Delta t_1} \\ \vdots \\ p_r e^{s_r (2N-1)\Delta t_1} \end{Bmatrix}} = \underset{2N\times 1}{\begin{Bmatrix} p_r e^{s_r 0\Delta t_1} \\ p_r e^{s_r 1\Delta t_1} \\ \vdots \\ p_r e^{s_r (2N-1)\Delta t_1} \end{Bmatrix}} e^{s_r \Delta t_1} \quad r = 1, 2 \ldots 2N \tag{17}$$

This equation can be arranged to have a standard eigenvalue problem from which natural frequencies and damping ratio can be calculated from the value of s_r.

$$[\boldsymbol{A_s} - e^{s_r \Delta t_1}\boldsymbol{I}]\begin{Bmatrix} p_r e^{s_r 0\Delta t_1} \\ p_r e^{s_r 1\Delta t_1} \\ \vdots \\ p_r e^{s_r (2N-1)\Delta t_1} \end{Bmatrix} = \boldsymbol{0} \tag{18}$$

For underdamped case:

$$\begin{aligned} s_r &= a_r \pm b_r \\ &= -\xi_r \omega_r \pm i\omega_r \sqrt{1-\xi_r^{\,2}} \end{aligned} \tag{19}$$

The relationship between the eigenvalues (which is in complex form, $\beta_r + i\gamma_r$) and natural frequency are as follow:

$$\begin{aligned} f_r &= \frac{|\ln(\beta_r + i\gamma_r)|}{2\pi\Delta t} \\ &= \frac{|a_r + b_r|}{2\pi} \end{aligned} \tag{20}$$

For damping ratio:

$$\begin{aligned} a_r &= -\xi_r \omega_r \\ \xi_r &= \frac{-a_r}{\omega_r} \end{aligned} \tag{21}$$

In order to confidently identify the natural frequency and damping ratio of each mode of the system, a technique known as '*Modal Confidence Factor*' (MCF) [7] is applied to distinguish between computational and genuine modes as N is increased.

If p_{ij} is the i^{th} element of the j^{th} identified eigenvector or j^{th} mode at the actual measurement, then the same measurement delayed at $\Delta t_3 = 10\Delta t_1$ is expected to be:

$$\overline{p_{ij}} = e^{\lambda_j \Delta t_3} p_j \quad (22)$$

if this identified eigenvector is a structural mode [7, 8]. Hence, MCF can be defined as the ratio between left and right hand side of (22) as below:

$$\begin{aligned} MCF_{ij} &= \left| \frac{e^{\lambda_j \Delta t_3} p_j}{\overline{p_j}} \right| \quad for \quad \overline{p_j} > e^{\lambda_j \Delta t_3} p_j \\ &= \left| \frac{\overline{p_j}}{e^{\lambda_j \Delta t_3} p_j} \right| \quad for \quad e^{\lambda_j \Delta t_3} p_j > \overline{p_j} \end{aligned} \quad (29)$$

and should be near unity for genuine mode.

Modified SSTD Algorithm

In SSTD algorithm the time signal responses is used to extract modal parameters from these responses as standard eigen value problem. This can be achieved by manipulating the time interval from the data gathered as is observed mathematically from references [2] and [5]. The effect of different intervals for time shift towards the accuracy of natural frequencies and damping ratio was investigated. Following cases were considered for both matrices, [X] and [X^] where the time interval is shifted with:

1. multiple interval by 1 (default settings in the algorithm)
2. multiple interval by 2
3. multiple interval by 3
4. multiple interval by 4
5. multiple interval by 5

Analysis and Procedures

The analysis was carried out with Matlab using five generated data sets for a simple beam structure with known properties as summarised in Table 1 and Table 2 below. These data sets with known degree of freedom are tampered with 'white Gaussian noise' value with signal to noise ratio (SNR) equals to 100. The samples of the data sets are as shown in Figure 1 to Figure 3. The reason for using 5 different sets of data is to resemble the response of a system at 5 different locations when a single input is applied at any one of these points. All the simulated data will be simultaneously processed using the algorithm by means of least square approach as expressed in [14] and [15]. The selection of convergence of the natural frequencies and damping ratios were based on MCF value greater than 0.9 [16,17]. The percentage errors for these parameters were then calculated to make assessment towards the accuracy of natural frequency and damping ratio for each case introduced.

The details for simulated data are summarized below:

Table 1: Data Collection Parameters

Sample rate	2048 Hz
Number of samples	2048
Frequency resolution	1 Hz
Nyquist frequency	1024 Hz

Table 2: Properties of the Simulated Data Set

Mode	Residue	Natural Frequency	Damping Ratio
1	10	128.0	0.00300
2	20	256.0	0.00250
3	30	512.0	0.00139
4	39	768.0	0.00093
5	50.7	806.4	0.00046

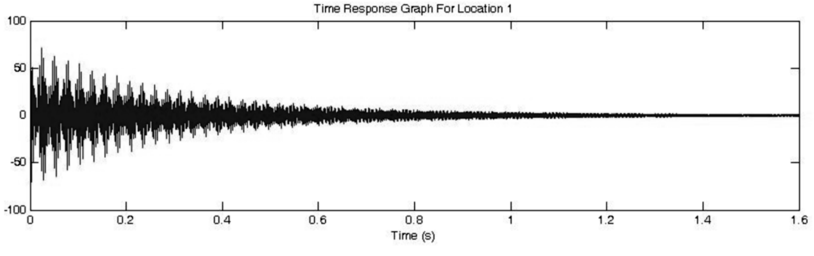

Figure 1: One of the Time Response Plot that was Corrupted by White Gaussian Noise of SNR = 100

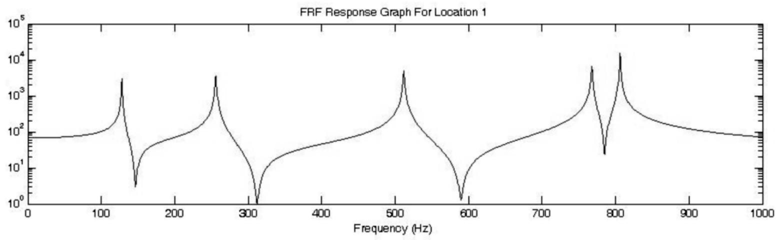

Figure 2: One of the FRF Plot that was Corrupted by white Gaussian Noise of SNR = 100

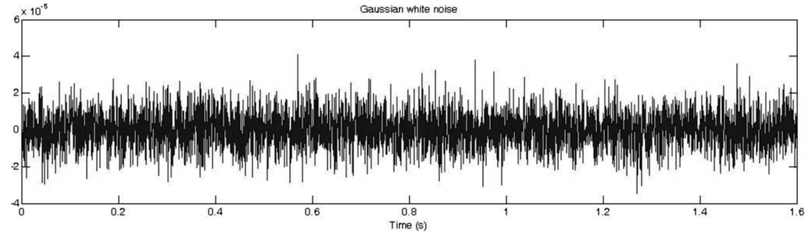

Figure 3: Signal of white Gaussian Noise with SNR = 100 was Applied for Each Data Set

Results and Discussion

The results of the five cases for each set of data are depicted in Figure 4 to 9. From the results, it can be observed that Case 5 gave the most inaccurate and inconsistent results among all the cases investigated. It is important to clarify the usage of percentage error for natural frequency and damping ratio. As an example, in Case 5, the percentage error of natural frequency for mode 4 was nearly 3%. This value means that if the real value for this parameter at the same mode stated in Table 2 as 768 Hz, 3% error would mean that the frequency is approximately either 790 Hz or 745 Hz. On the other hand, for damping ratio in Case 5, error of merely 300% as reported at mode 2 and 4 mean that it can be either -0.6 (which is not desired as mathematically it promotes instability) or 1.2 if the true value is 0.3. From this reason, a certain limit of error acceptability must be defined in order to make decision whether to accept or to reject the results. These limits must also be different as same value of percentage error gives different definition to both parameters. This indirectly gives a small conclusion that as the value of damping ratio is very small if compared to natural frequency, small deviation in a very small number would produce larger percentage error even though the deviation is the same in both parameters.

For this analysis, it is defined that deviation as much as 4 Hz from true value is used as the limit while for damping ratio, maximum data deviation is 0.15

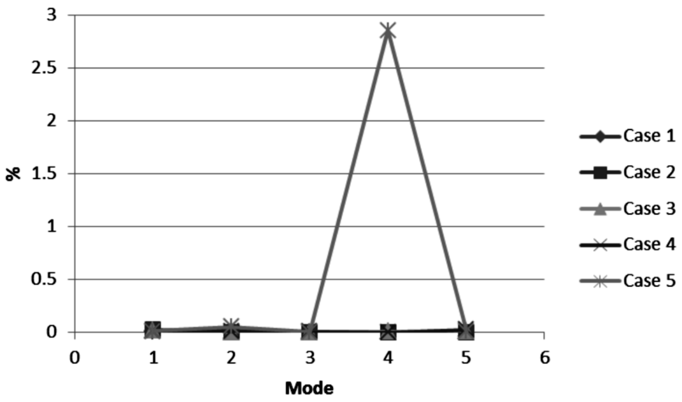

Figure 4: Simulated Data Results: Natural Frequency Percentage Error for Five Cases

will be applied. In other words, maximum percentage error interpreted for this analysis for natural frequency is 0.26% and 50% for damping ratio. Any value greater than these is considered erroneous. Thus further analysis will focus to Case 1 to 4 only.

In terms of accuracy, Case 1 has the most consistent result throughout all modes which refers to low percentage error at all modes. On the contrary, Case 4 highlights the most consistent results in terms of keeping low percentage error at each mode in comparison to other cases. The most significant result achieved with Case 4 can be seen at mode 5 with percentage error of 2.73% while the next lowest percentage is produced by Case 1 which is 26.57%. Although Case 4 has the highest percentage error for natural frequency at the same mode, the value of that error is considered very small for assessment of natural frequency. In practical, this figure means that it has 806.62 Hz while the true value is 806.4Hz. Besides, more interest lie at how to reduce the percentage error for damping ratio as the parameter is much more challenging as discussed previously. This is also illustrated by observing together Figure 4 and Figure 5 as well as Figure 6 and Figure 7.

The other issue is maintaining the result distribution acquired by the algorithm as the number of identifications order, N is increased so that the standard deviation of the results for that mode is low. When the standard deviation is low, this will produce more accurate value of mean damping ratio. In order to explain this phenomenon, Figure 8 and Figure 9 show the results distribution for damping ratio for Case 1 and 4 only for mode 5.

When N is increased, modal parameters which are natural frequency and damping ratio are calculated for that N. However, due to the present of noise, the calculation may produce computational values of both parameters at that N. In order to remedy the problem, MCF is introduced to distinguish between the

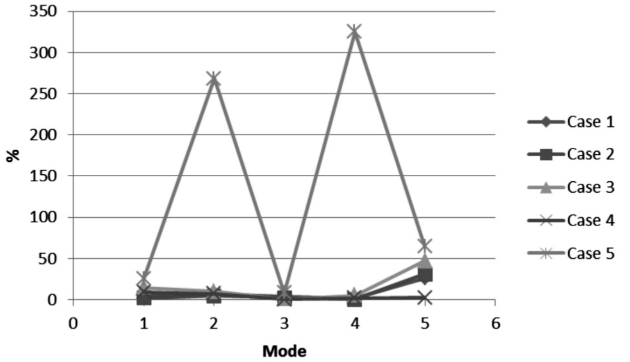

Figure 5: Simulated Data Results: Damping Ratio Percentage Error for Five Cases

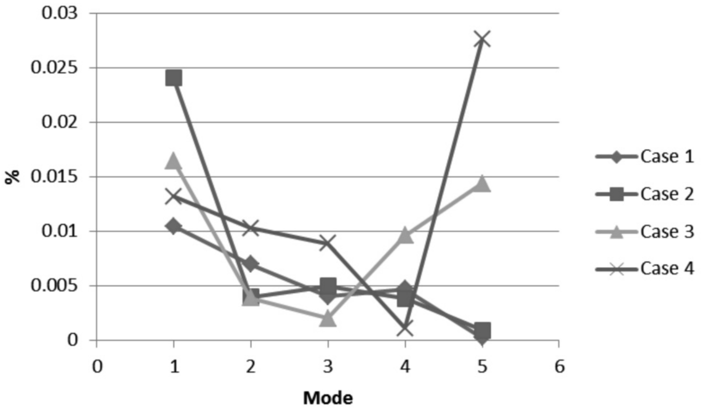

Figure 6: Simulated Data Results: Natural Frequency Percentage Error for Four Cases

genuine and computational parameters. To add more confidence, *N* is increased so that series of genuine modal parameters would appear consistently for each *N* thus indirectly indicates the genuine parameters for that system. In modal analysis field, these results are plotted upon the FRF and the plot is known as stability diagram. Figure 10 portrays the stability diagram of natural frequency for Case 1 throughout all modes.

Figure 8 and Figure 9 show the damping ratio distributions for each *N*. From this figures, it can be seen that the results distribution for Case 4 are closer to the true value than Case 1. These figures also explains that one of the factor affecting the calculation of mean damping ratio is due to some data that are located far from the mean. With their presents would certainly give effect to the calculated mean damping ratio. Hence, the algorithm can be improved by introducing filters to identify these outliers and ignore them during the calculation of the mean.

The other aspect that relates to the accuracy of results is the number of data calculated by the algorithm. Figure 11 shows the number of data produced by each case for each mode. From the graph, it is understandable why the results of damping ratio for Case 5 at mode 2 and 4 were erroneous. It was due to insufficient number of data that can provide enough information to describe that parameter of the system.

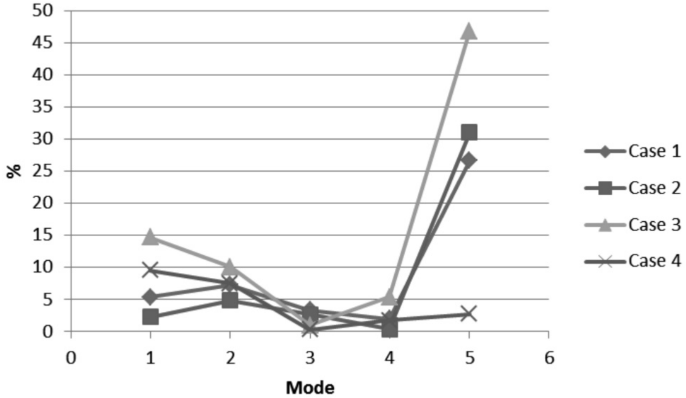

Figure 7: Simulated Data Results: Damping Ratio Percentage Error for Four Cases

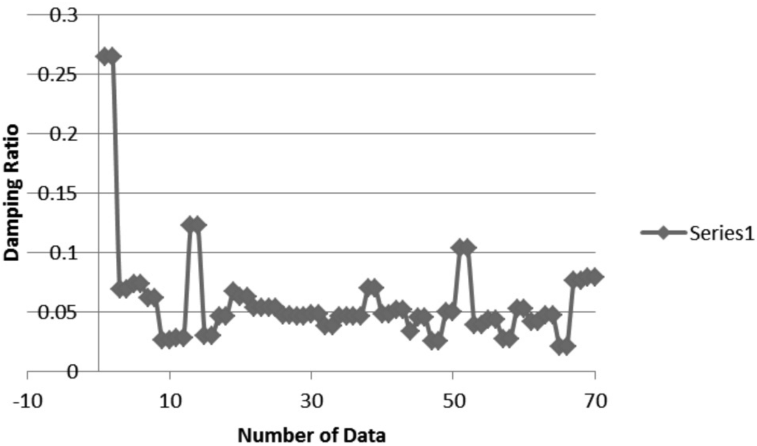

Figure 8: Damping Ratio Distribution for Case 1

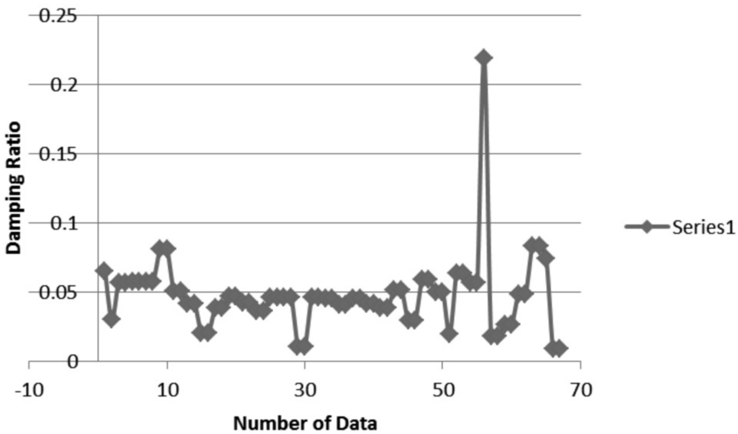

Figure 9: Damping Ratio Distribution for Case 4

The results have shown that all cases except case 5 provide acceptable results for natural frequencies and damping ratios determination. It can be further deduced that from Figure 11 that enough data points are available to ensure sufficient information for extraction of accurate modal parameters value. From the Figure 11, minimum number of data required to produce sufficiently accurate result is approximately 40 data points. Although by the amount of data of 20 is sufficient to calculate natural frequency as shown by Case 5, this amount is not sufficient enough to accurately identify damping ratio. However,

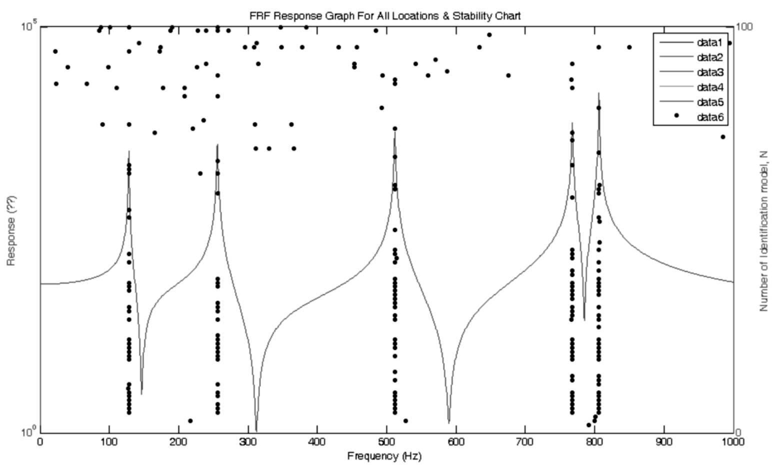

Figure 10: Stability Diagram of Natural Frequency for Case 1

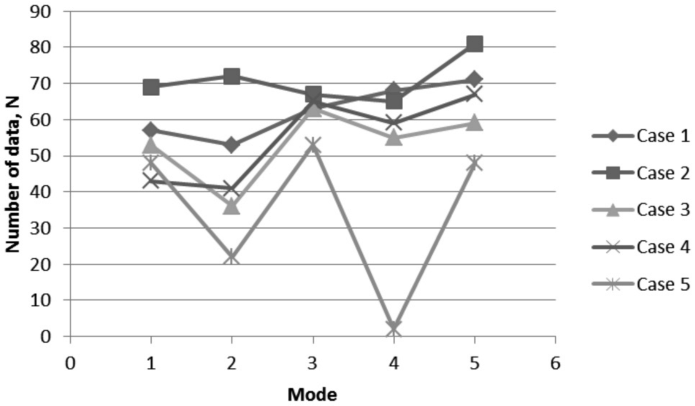

Figure 11: Number of Results Calculated for Natural Frequency and Damping Ratio for Five Cases

this conclusion is still need to be further tested with other case study in order to verify this statement.

Other conclusion can be drawn here is that making the interval bigger does not mean that the number of data produced will decrease although the overall trend shows that as the interval gap is increased, less data are produced. Such evidence is depicted by Case 2 where the number of data is larger than original Case 1. More importantly, the issue of having the sufficient amount of data that is able to provide sufficient information regarding dynamic characteristics of the system. If the data is large but not sufficient to give dynamic characteristics, the data is useless as the objective of doing modal analysis is to accurately identify modal parameters. This is an explanation on why results for Case 4 are better and accurate than Case 1 although the data for both cases are taken from the same source. Indirectly, this refer to the presence of noise in data which has made some of the data to produce values of modal parameters that is located far from the majority of other values and hence affected the mean value of these parameters. When the interval in Case 4 is bigger than original Case 1, some of the data taken by Case 4 were neglected at which these neglected data were the data that could possibly lead to outliers.

Figure 12 illustrates the time taken for each case to calculate all the modal parameters. From the graph, it can be seen that as the interval is bigger by multiple *n*, the time taken is shorter. Hence, if in Case 4 can produce acceptable results for natural frequency and more accurate results for damping ratio than the original SSTD settings which is Case 1, it will be time saving especially for larger data. The table below summarizes the results for Case 1 and Case 3 for both data sets.

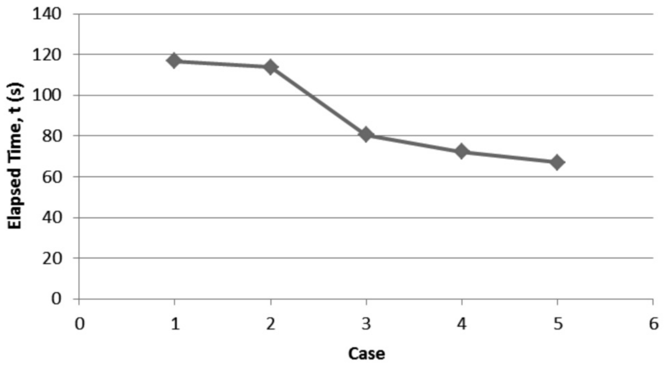

Figure 12: Time Taken to Calculate Natural Frequency and Damping Ratio for Five Cases

Table 3: Percentage Error Results for Natural Frequency

Case	Mode				
	1	2	3	4	5
1	0.01045	0.00696	0.00402	0.00471	0.00026
2	0.02408	0.00397	0.00497	0.00385	0.00092
3	0.01645	0.00389	0.00203	0.00963	0.01440
4	0.0132	0.01031	0.00887	0.00111	0.02757
5	0.0132	0.05517	0.00204	2.85163	0.00788

Table 4: Percentage Error Results for Damping Ratio

Case	Mode				
	1	2	3	4	5
1	5.38418	7.20952	3.33986	1.95477	26.57179
2	2.21516	4.84659	2.63524	0.34905	30.95950
3	14.64928	10.01151	0.93438	5.32636	46.79995
4	9.49323	7.47539	0.25137	1.74766	2.73306
5	25.47262	268.21743	8.33777	324.79888	64.48306

Conclusion

In this study, the modified SSTD technique with varying time shift interval was carried out successfully to improve the accuracy of natural frequencies and damping ratios determination using simulated data sets. It is shown that certain time interval setting influences the natural frequencies and damping ratios accuracy prediction. However, further tests on actual experimental results are required to ensure the validity and accuracy of this technique before being put to practice.

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