Developing New Lab Base Teaching Approach for Linear Algebra subject in Engineering Mathematics Courses

N. Lohgheswary[*]*, M. H. Abdul Halim*
Faculty of Engineering and Built Environment, SEGi University

Z. M. Nopiah
Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia

A. A. Aziz
Faculty of Computing and IT, KingAbdulAziz University

E. Zakaria
Faculty of Education, Universiti Kebangsaan Malaysia

[*]*lohgheswarynagarethinam@gmail.com*

ABSTRACT

The underachievement of engineering students in engineering mathematics is due to poor prior knowledge and anxiety towards mathematics. This study aim to develop a new lab based teaching approach for engineering mathematics students in Universiti Kebangsaan Malaysia (UKM) via computing applications. This has called for the investigation of the curriculum of engineering mathematics or in specific, the syllabus of the content of Linear Algebra in UKM. The Linear Algebra syllabi in 61 universities were compared with the UKM's 14 weeks' Linear Algebra syllabus. The eight highest percentages of common topics in Linear Algebra were identified. Matrix and basic operations, application in linear system, vector space and subspace, eigen value and eigen vector, diagonalization and series are the eight topics which will be taught in lab sessions in computing language. To justify these findings, many researchers have been using various computing tools such as Mathcad, Maple and MATLAB to

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teach Linear Algebra in the past ten years. Positive feedback received from students and this new teaching approach help them to understand the subject well. Embedding computing applications in teaching Linear Algebra is an innovative way to teach students. Therefore it is hoped that this new teaching method will improve UKM's engineering students' grades as well as give exposure to a new teaching and learning environment.

Keywords: *Engineering mathematics, Linear algebra, Syllabus, Lab sessions, teaching and learning*

Introduction

Engineering Mathematics is the foundation of all other engineering courses. Vector Calculus, Linear Algebra and Differential Equations are the common subjects in Engineering Mathematics for all of the engineering courses at university level. A good learning method will ensure higher performance among university students.

Recently, some researchers have commented on poor achievement among Engineering Mathematics subjects. Underachievement is mainly related to poor prior knowledge of mathematics. Students with poor prior knowledge face more difficulties in understanding at the tertiary level of engineering mathematics. This is because the content of engineering mathematics especially Linear Algebra is highly abstract. Students will eventually tend to feel anxious in learning engineering mathematics.

Tang *et al.* [1] related students' underachievement in mathematics courses with influencing factors. Questionnaire related to lecturers' opinions, final examination report analysis and students' details are collected from the registration database. Pre-Calculus, Calculus I, Mathematics II and Engineering Mathematics I were identified as underachievement courses. Class size and gender were two factors for poor performance in these subjects.

Borba [2] commented from an online news forum that 66 percent of students from Fresno State who enrolled in Math 75 which is known as Calculus I failed the subject. The reason behind this higher percentage is the lack of preparation from high school, students' declining study habits and the course itself. Besides, students' attitude of forgetting algebra from high school and students' anxiety on Calculus are the other reasons for the poor grades. Statistics show that 141 out of 316 Math 75 students only obtained grades lower than C in the subject. This figure also includes 47 students who withdrew from the Math 75 course.

Hailikari *et al.* [3] explored some alternative ways to prior knowledge and related them to students' achievement in a mathematical case study. This

study investigates how various types of prior knowledge influence students' achievement. The study consists of a sample of 202 mathematics students who sat for the prior knowledge test during the first lesson. The final grade of the course was taken as the students' achievement. The study revealed that the type of prior knowledge makes a difference. The model that assesses the procedural knowledge was reflected on the final grades. In addition, the success from the previous student was found to be the best predictor of student achievement.

Gynnild [4] showed evidence that Maths performance of students in secondary school was related to the performance of basic Calculus course in university. Poor performers in Maths in secondary school are also poor performers in the Calculus course. The study also gives some additional information that students tend to enter the university with two motivations, one to increase their career opportunity and the other to study the subjects they are interested in. This study further illustrates three major reasons for students' underachievement. The study approach, less effort and lacking of skills in the math entry are the factors for poor performance. This is classified as 'students' responsibility' towards learning. On the other hand, lecturers and tutors should review the learning objectives for the target group. This includes the selection of the teaching content, estimated work load, ways of assessment and strategies for lecturing and tutoring.

Ma and Xu [5] commented that poor achievements in Mathematics were significantly related to the subsequent later high level anxiety. Mathematics anxiety is one of the effects of poor achievement in Mathematics. This includes dislike, worry, fear, tension, distress, helpless and mental disorganization.

Poor achievement in Engineering Mathematics has spurred the review of the Linear Algebra syllabus in Universiti Kebangsaan Malaysia (UKM), a public university in Malaysia. A new learning method should be introduced to give a better understanding in the procedural knowledge of the subject. On the other hand, this will reflect positively in students' achievement in all the Engineering Mathematics subjects.

Methodology

All the engineering students in UKM will be introduced to Linear Algebra subject, the second Engineering Mathematics subjects in the second semester of study. The pre-requisite for this subject is Vector Calculus. Differential Equations is the next common Engineering Mathematics subject after Linear Algebra. All the three subjects are common for students from departments of Electrical, Mechanical, Chemical and Civil engineering. The detail of Linear Algebra's weekly syllabus for UKM is listed in Table 1.

Table 1: UKM's weekly syllabus of Linear Algebra

Week 1	Syllabus
1	Matrix and basic operation. Gaussian Elimination, Determinant-cofactor Expansion, Row Reduction and Cramer's rule, Inverse Matrix.
2	Applications: Linear System and Leontif Input-Output models.
3	Vector Space and Subspace. Linear independence, Basis, Dimension. Change of Basis.
4	Row Space, Column Space, Null space, Rank, Nullity.
5	Matrix Transformations: Applications: Geometry of Matrix Operator, Dynamical Systems and Markov Chain.
6	Eigen value, Eigen vector. Application- Differential Equation.
7	Inner Product Spaces- Gram-Schmidt Process. Applications- Least Square Fitting and Function Approximation.
8	Diagonalization and Quadratic Forms.
9	Linear Transformation- Isomorphism, Composition and Inverse Transformation.
10	LU-Decomposition, Power Method. Applications-Numerical Method.
11	Sequence, Infinite series, Partial Sum, Types of Series: Geometric Series, Harmonic series, P-series, Alternating series.
12	Convergence- Integral Test, Comparison Test, Ratio Test, Root Test.
13	Power series, Radius of Convergence, Taylor series, MacLauren series.
14	Fourier series, Cosine and Sine series.

The weekly Linear Algebra syllabus of UKM is compared with 61 world top universities. 20 universities are selected from the United States. They are Massachusetts Institute of Technology, Stanford University, University of California, California Institute of Technology, Princeton University, Georgia Institute of Technology, Carnegie Mellon University, University of Texas at Austin, University of Michigan, Cornell University, University of Illinois at Urbana Champaign, Northwestern University, University of Wisconsin_Madison, Columbia University, University of Washington, University of Minnesota, Rice University, Purdue University, Ohio State University and Pennsylvania State University.

20 universities are selected from the United Kingdom. They are University of Cambridge, University of Oxford, Imperial College London, University of Manchester, University College London, University of Edinburgh, University of Nottingham, University of Bristol, University of Southhampton, University of Leeds, University of Sheffield, University of

Liverpool, The University of Warwick, University of Bath, University of Strathclyde, Cardiff University, New Castle University, Queen Marry University of London, University of Glassgow and University of Surrey.

10 universities from Oceania are choosen for this comparative study. They are University of Melbourne, University of Queensland Australia, University of Sdyney, Monash University and University of New South Wales, University of Auckland, University of Otago, University of Canterbury, Victoria University of Wellington and Massey University.

In addition, 6 universities from Asia is taken for comparison study. They are National University of Singapore, Nanyang Technological University, Hong Kong University of Science and Technology, The University of Hong kong, Kyoto University and Seoul National University.

5 universities from Malaysia are chosen too for this study. They are Univeristi Malaya, Universiti Teknologi Malaysia, Universiti Sains Malaysia, Universiti Putra Malaysia and Universiti Teknologi Mara.

Firstly, every university's website was explored to get into the engineering faculty and engineering department. Then the list of engineering mathematics subjects offered is identified. Then the subject syllabus was compared to UKM's Linear Algebra syllabus content. If the university syllabus has the same weekly topic as UKM, 'x' will be marked in the Table 2 and Table 3.

Next the total 'x' for 61 universities for week 1 was calculated. For example 'Matrix and basic operation' is common for 49 universities. Then the percentage for that topic is being calculated in the following way.

$$\frac{49}{61} \times 100 = 80\%$$

Under the same method, the other 13 weekly topic percentages have been calculated.

Results and Discussion

Table 2 and Table 3 show the matching of the UKM's and 61 universities Linear Algebra syllabus. Table 4 shows the percentages for the weekly topic for Linear Algebra subject.

Table 2: Matching of UKM and other universities Linear Algebra syllabus

WEEK / UNIVERSITY	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	X	X	X	X	X	X	X	X		X	X	X	X		X	X		X	X	X	X		X	X		X	X			X
2	X	X		X	X		X				X	X			X				X	X					X					X
3	X		X	X		X	X	X		X	X	X	X		X		X	X	X	X		X	X			X	X			
4	X			X		X	X				X		X		X				X	X		X					X			
5	X	X	X	X	X	X				X	X							X	X				X		X					X
6	X	X	X	X	X	X	X	X		X	X	X	X		X	X	X	X	X	X	X		X		X	X	X		X	X
7	X		X	X		X		X		X	X		X		X						X									
8	X	X	X	X	X	X		X		X	X		X		X			X	X		X	X	X		X	X				X
9	X												X		X				X			X								
10	X				X																X									
11							X	X	X					X										X			X	X		
12							X	X	X					X										X			X	X		
13						X	X	X	X					X										X		X	X	X		
14	X						X																			X	X	X	X	X

Table 3: Matching of UKM and other universities Linear Algebra syllabus

WEEK / UNIVERSITY	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**
1	X	X	X	X	X	X	X	X	X		X	X	X	X	X		X	X	X	X	X	X	X			X	X	X	X	X	
2		X					X	X			X		X	X	X			X			X		X			X	X			X	
3							X		X		X	X	X		X	X	X				X	X	X	X		X	X	X			
4									X		X	X	X		X						X		X			X	X				
5						X						X			X	X	X									X					
6		X		X	X	X	X	X	X		X	X	X	X	X		X	X	X	X	X		X	X	X	X	X	X	X		
7			X												X		X				X		X			X					
8						X							X		X		X				X		X				X				
9											X						X				X		X								
10																													X		
11		X				X				X	X				X		X	X	X									X			X
12						X					X			X	X		X	X	X			X					X				X
13	X		X		X	X				X	X	X	X	X	X		X	X	X								X	X	X	X	X
14	X			X		X				X	X		X	X	X	X	X		X		X	X	X		X				X	X	X

Table 4: Percentage of weekly topic for Linear Algebra

Week	Total	Percentage
Week 1	49	80
Week 2	25	41
Week 3	34	56
Week 4	20	33
Week 5	19	31
Week 6	49	80
Week 7	16	26
Week 8	26	43
Week 9	9	15
Week 10	4	7
Week 11	17	28
Week 12	17	28
Week 13	27	44
Week 14	25	41

Figure 1 shows the distribution of weekly topics for Linear Algebra in a bar chart.

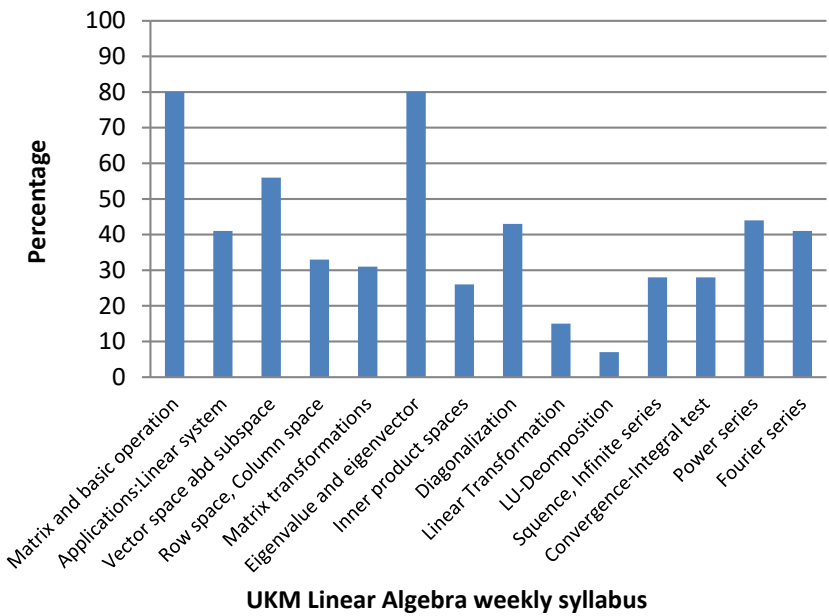

Figure 1: Distribution of weekly topics for Linear Algebra

For Linear Algebra subject, eight topics with highest percentage is identified as common topics for UKM and other universities. They are topics from weeks 1, 2, 3, 4, 6, 8, 13 and 14. The matrix and basic operation, application in linear system, vector space and subspace, row space, column space, eigenvalue and eigenvector, diagonalization, power series and fourier series are the common topics in Linear Algebra.

Since the last ten years , in a decade, some researches have been using Computer Algebra Sysytem (CAS) to teach Linear Algebra. This is an alternative method of teaching compared to traditional lectures. Charney [6] found that Virgina Tech students were weak in the Matrix Methods of Structural Analysis course. The course needs Theory of Structures and basics of Linear Algebra. Students found difficulties in coping with this subject as their knowledge is below par in programming. To overcome the students' difficulties, a course namely Computer Methods of Structural Analysis I was introduced. Mathcad was incorporated in the course as it was simple to learn and thus it will give a strong foundation in the Matrix Structural Analysis. Since Mathcad is highly visual, it was used as a visual matrix manipulation tool and to write structural analysis programs. This subject requires the students to comprehend well the overall program and it requires students to implement the theory as well. One of the famous examples in Complex structure, the cantilevel beam could be easily analyzed by integrating it in Mathcad but it cannot be analyzed manually.

Kilicman *et al.* [7] applied Maple software in nine tutorials in Linear Algebra which includes solving the linear system, plotting eigenvectors and computing eigenvalues. Maple reduces the level of difficulties in subjects such as Real Analysis and Applied Mathematical Courses which includes applications of eigenvalues and eigenvectors. Students compared Maple solution with the hand-written solution to correct their work. Technological tools helped students to learn Linear Algebra materials in a detailed and resourceful manner. During the problem solving process, students were able to apply the concepts well as their skills were improved with Maple. In short, Maple helps to enhance Learn Algebra concepts both theoretically and computationally.

Maple was introduced to the first year engineering mathematics course, Linear Algebra at The Technical University of Denmark. Schmidt *et al.* [8] discussed the advantages and disadvantages of implementing the technological tool. Maple was viewed as a tool for visualization in lectures. Maple was used in designing group exercises as well as in projects. From 2007 to 2008, surveys were in array to find the relationship between study resources and study activities. Study resources include textbooks, the Maple Demos, Internet and course materials while lectures, group exercises, quizzes and homework are classified as the study activities. In week three, nine and nineteen of the semester, students' feedback was recorded. There are some

positive changes that occured. Students' participation in group exercise was positive and they spent more time in completing homework. However, the disadvantages are that students' class attendance was low and students preparation for classes was low. Maple Demos makes use of the textbook as its study resources as students achieve better grades in the Maple session.

Van *et al.* [9] created an intelligent educational software system to solve problems in Linear Algebra. The three areas of concentration are Linear Equations System-Matrix-Determinant, Vector Space and Linear Map. The coding was implemented in Maple and the solutions were designed using C programming language. The students from University of Informational Technology tested this intelligent educational software system. Students commented positively stating that it is highly useful to study Linear Algebra.

Chen [10] introduced MATLAB to Linear Algebra students. Linear Algebra was categorized as a subject with high abstract and theories. Thus students face problems in absorbing the knowledge and apply the theory. Students ended bored as more time was wasted in solving complex questions through handwriting. Students managed to obtain fast results with high accuracy using MATLAB. MATLAB was able to visualize the abstract Linear Algebra concepts. This computing tool can boost students' interest towards learning the subject. It also enhance students' learning methods both in theory and application.

As these researches found that introducing software in teaching and learning of Linear Algebra, thus eight lab sessions is suggested to be introduce to the UKM students to help them to understand the subject better. Each lab session should be conduct for two hours. Table 5 illustrates the details of lab sessions which is extracted from Figure 1.

Table 5: Lab sessions

Week 1	**Syllabus**
1	Matrix and basic operation. Gaussian Elimination, Determinant-cofactor Expansion, Row Reduction and Cramer's rule, Inverse Matrix.
2	Applications: Linear System and Leontif Input-Output models.
3	Vector Space and Subspace. Linear independence, Basis, Dimension. Change of Basis.
4	Row Space, Column Space, Null space, Rank, Nullity.
5	Eigen value, Eigen vector. Application- Differential Equation.
6	Diagonalization and Quadratic Forms.
7	Power series, Radius of Convergence, Taylor series, MacLauren series.
8	Fourier series, Cosine and Sine series.

Many of these researchers have conducted lab sessions via different computing application tools for the suggested lab topics in Table 5. Mathcad, Maple and MATLAB are among the software used to teach Linear Algebra. This new method of teaching has shown positive results and feedback among students.

From Table 5, lab for week one encompasses matrix and basic operations. The syllabus covers Gaussian elimination, determinant-cofactor expansion, row reduction, Cramer's rule and inverse matrix. Charney [6] who conducted Mathcad classes in teaching the matrix method for the matrix structural analysis subject agreed that the cantilevel beam in the complex structure could not analyzed without having it integrated in MAthcad. Van et al. [9] created educational software to solve matrix-determinant problems via Maple.

Second week lab concerntrates on applications in the linear aystem and leontif input-output models. KIlicman et al. [7] applied Maple in solving the linear system. Besides Van et al. [9] too had solved the linear equations system via Maple.

Lab three covers row space and subspace, linear independence, basis, dimension and change of basis. Row space, column space, null space, rank and nullity are suggested in labs for week four. Educational software system via Maple by Van et al. [9] includes topics of vector space. Students' feedback were positive as it was very useful for them to study linear algebra.

Lab for week five covers topics namely eigen value, eigen vector and also the application in differential equation. Kilicman et al. [7] concluded that the inclusion of Maple in real analysis and applied mathematics courses reduced the difficulties in plotting eigen vectors and computing eigen values.

Next, diagonalization and quadratic form are the suggested topics for week six. Power series, radius of convergence, Taylor series and MacLauren series are lab topics for week seven. Lastly fourier series, cosine and sine series are the lessons arranged for lab week eight.

Schmidt et al. [8] viewed Maple as a tool for visualization in linear algebra courses. Students were very positive in group participation and in completing their homework. Chen [10] too agreed that software especially MATLAB is a tool that can help visualize abstract linear algebra concept.

Since all the researchers' comments were very positive about the new method of teaching which is incorporating computing applications in teaching linear algebra therefore teaching linear algebra via computing application is suggested as a new teaching method for engineering mathematics students in UKM. It is hoped that UKM students will benefit well from this new teaching method.

Conclusion

Linear Algebra is a common Engineering Mathematics subject. It has high abstract content and if one does not understand the subject in-depth, it will be difficult to undergo the course. In the first step to assist the teaching of Linear Algebra, the 14-weeks syllabus content of Linear Algebra from UKM was cross-examined that of 61 world universities. The common topics identified are Matrix and basic operation, Application in Linear System, Vector Space and subspace, Eigen value and Eigen vector, Diagonalization and Series.

Next, a new learning method is introduced which is learning Linear Algebra with computational tools. The recent researches confirmed the impact of computational tools help students to visualize the abstract content and the students will be able to understand the procedural knowledge better when it comes to learning with computers. The impact anticipted with the incorporation of new lab base teaching method is to achieve the learning outcomes will lead to enhancement of students' performance in Linear Algebra subject and also any other engineering subjects.

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