UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

FOURTH HANKEL DETERMINANT FOR SUBCLASSES OF BOUNDED STARLIKE FUNCTION

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

Let B denoted the class of functions $f(z)$ which are analytic with an open unit disc $E = \{z : z \in R \text{ and} |z| < 1\}$ and satisfy

$$\text{Re}\{\alpha f'(z) + \beta z f''(z)\} > 0$$

for some $0 < \alpha \leq 1$ and $0 \leq \beta < 1$ where $f(z)$ is a normalized starlike function. One of the properties of class of function B can be determined by finding fourth Hankel determinant for this class of function. In order to find fourth Hankel determinant, there are a few steps will be used. The extreme function must be first determined for the function of a certain class in order to study the properties. Next, compare the coefficients of like powers of z, z^2, z^3 and z^4 between left hand side and right-hand side of function. Then, we obtain coefficient in terms of a_n and substitute the value of the coefficient in the functional. The researchers apply Lemma and use triangle inequality to solve the right-hand side of the function in order to get functional $\left| a_2 a_3 - a_3^2 \right|, \left| a_2 a_3 - a_4 \right|$ and $\left| a_3 - a_2^2 \right|$. After that, the function $F(c, \phi)$ will maximize on the closed region $[0, 2] \times [0, 1]$. Same procedures will be used to find third Hankel determinant for new class of function. Lastly, find Δ_1, Δ_2 and Δ_3 determinants. Then, apply triangle inequality in each determinant to get the expressions. After that, substitute into fourth Hankel determinant, $H_{4,1}(f)$ to obtain the result. The sharp upper bound result obtained for $0 < \alpha \leq 1$ and $0 \leq \beta < 1$, it is coincides to the previous study and this property leads to development of fourth Hankel determinant. Furthermore, researcher also can explore the existing class of function with other behaviour to getting better result. Lastly, since mostly previous researcher finding the result in a univalent function, thus we recommend other researcher to find the result in multivalent function.