

**UNIVERSITI TEKNOLOGI MARA**

**TECHNICAL REPORT**

**COMPARISON BETWEEN KELVIN-VOIGT  
MODEL AND MAXWELL MODEL IN  
VEHICLE-TO-BARRIER CASE**

**P27S19**

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## **ABSTRACT**

Accident is one of the most common incidents happen throughout the world. In the ASEAN countries, Malaysia is the top country with the highest risk of road death (per 100,000 population) and most of the road accidents cause mortality. Crashworthiness is related with the vehicle crashes where it is defined as the ability of the vehicle to protect itself from the effect of the accident. It is used widely by some vehicle manufacturers to implement their vehicle crash test. The test is very expensive and time consuming. Besides that, it requires a bigger space and specific device such as accelerometer to implement the test. In order to solve this problem, this research came out with two basic viscoelastic models which are Kelvin-Voigt model and Maxwell model. Both models are used to find the displacement, velocity and acceleration of vehicle crash. These criteria are used by vehicle manufacturers to rate their vehicle. This research not only focus on equation of motion, but also explaining and determining some important concept such as separation of time, coefficient of restitution and transition damping coefficient. The real data of vehicle crashes are collected from National Highway Traffic Safety Administration (NHTSA). The scope of this research only focuses on vehicle-to-barrier (VTB) case. At the end, the result shows Maxwell model is more accurate to the actual result since the value of root mean square error are lower than Kelvin-Voigt model for all kinematics response; displacement, velocity and acceleration.