Sliding Mode Controller Design Based on Pole Placement Method

*Nor Laili Mahmud**
Faculty of Engineering, University Selangor, Malaysia
Email: norlaili@unisel.edu.my

*Norhashim Mohd Arshad**
Ramli Adnan
Faculty of Electrical Engineering, UiTM Shah Alam, Malaysia
Email: norhashim@uitm.edu.my

ABSTRACT

Over the years, many researchers have considered the problem of stabilizing uncertain systems. In variable structure control (VSC) system, the controller structure around the plant is intentionally changed by using a viable high-speed switching feedback control to obtain a desired plant behavior or response. In this paper, the problem of designing a VSC law for uncertain system, Electrohydraulic control system is considered. Using Pole Placement approach, an alternative design method of a linear sliding surface which is linear to the state is developed. A sufficient condition for the existence of linear sliding surface is given and an explicit formula of linear sliding surfaces guaranteeing the quadratic stability of the reduced-order equivalent system dynamics restricted to the sliding surfaces is derived. The sliding mode controller is then applied to an Electrohydraulic servo system. The simulation works were performed using MATLAB/SIMULINK software. A comparison for the controller design using Pole Placement Method with and without the sliding mode control shows that the Pole Placement method with a sliding mode control method produces a better performance response.

Keywords: *Sliding Mode Control; Pole Placement Method; Electrohydraulic System; Uncertain System; VSC (Variable Structure Control).*

ISSN 1823-5514, eISSN 2550-164X
Received for review: 2016-04-29
Accepted for publication: 2016-11-24
Published: 2017-06-01

Introduction

Electrohydraulic control system is a complex system with regard to nonlinearity. The linearization based method has been suggested as an effective way of using the nonlinear model of the system in the control law. However, the linearized model is an approximation of the real system dynamics. The latter having uncertainties, the sliding mode controller (SMC) is then preferred because of its robust character and superior performance.

SMC design involves two crucial steps; the first phase is to design a set of sliding manifolds so that the system state restricted to them has desired dynamics, which is of lower order than the original systems. The second phase is to design switching feedback control so that the system state trajectories can be attracted to the designed sliding manifold in finite time and maintained on the manifold [1].

Electro-Hydraulic Servo System

The electro-hydraulic servo system which is the object of this study is composed of a double-ended hydraulic cylinder driven by a direct drive servo-proportional valve as shown in Figure 1.

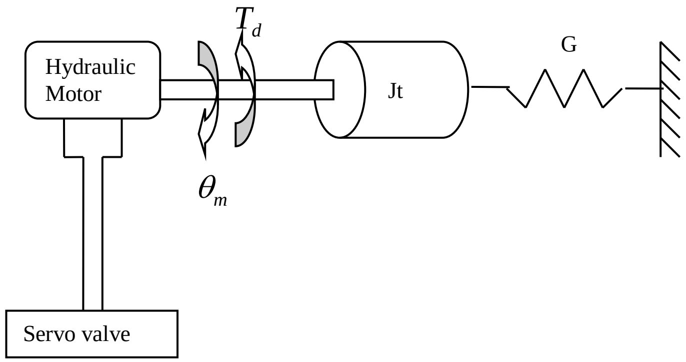

Figure 1: Physical model of a servo Electro-hydraulic control system

The dynamics equation for the electrohydraulic servo can be rewritten in state space form as:

$$\dot{X}(t) = AX(t) + BU(t) + W(X,t) \tag{1}$$

Or

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_{31} & -a_{32} & -a_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_{31} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ w_{31} \end{bmatrix} \tag{2}$$

Where the last row elements of matrices *A*, *B* and *W* are as follows:

$$a_{31}(t) = \frac{4\beta_e K_L G}{J_t V_t} \tag{3}$$

$$a_{32}(t) = \left(\frac{4\beta_e D_m^2 + 4\beta_e K_L B_m + G V_t}{J_t V_t} \right) \tag{4}$$

$$a_{33}(t) = \left(\frac{4\beta_e K_L J_t + B_m V_t}{J_t V_t} \right) \tag{5}$$

$$b_{31}(X,t) = \frac{4\beta_e D_m K_q(X) K_v}{J_t V_t} \tag{6}$$

$$w_{31}(X,t) = -\left(\begin{array}{l} \dfrac{4\beta_e K_L G G_n X_1^3(t) + 3 G G_n V_t X_1^2 X_2(t)}{J_t V_t} + \\ \dfrac{226.124\,\beta_e K_L X_1(t) + 56.526 V_t X_2(t)}{J_t V_t} \end{array} \right) \tag{7}$$

where

$K_c + C_1 = K_L$; the total leakage coefficient of the hydraulic system.

β_e	is the effective bulk modulus of the system
K_c	is the flow pressure coefficient
C_1	is the total leakage coefficient of the motor
J_t	is the total inertia of the motor and load
V_t	is the total compressed volume
D_m	is the volumetric displacement

B_m	is the various damping
K_q	is the valve flow gain
X_v	is the displacement of the spool in the servo valve
$GG_n\theta_m^3$	is the nonlinear stiffness term of the spring
G	is torsional spring gradient of the load

Electrohydraulic actuators are widely used in industrial applications. However, the control of hydraulic system is difficult because of its nonlinear dynamics, load sensitivity and parameter uncertainties [2]. The Simulink model for Sliding Mode Controller using Pole Placement method is shown in Figure 2.

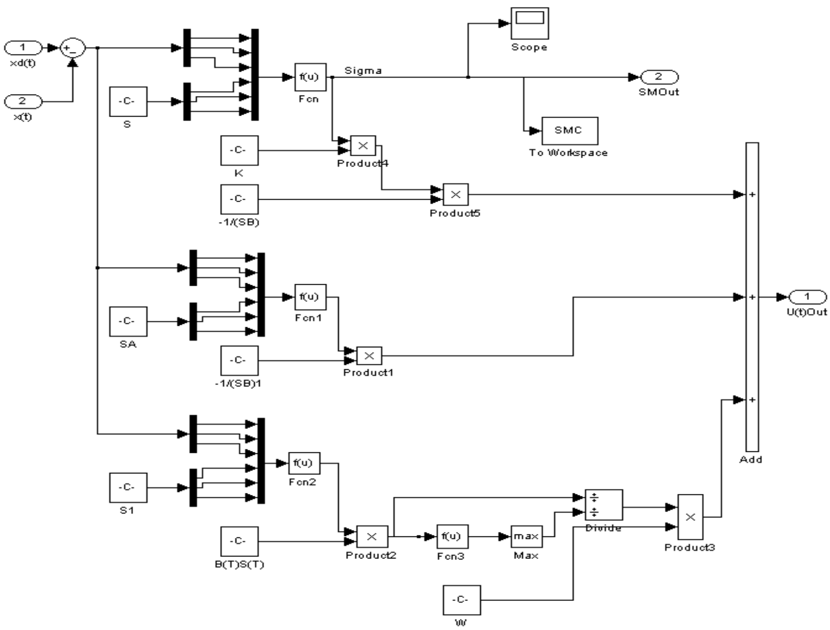

Figure 2: Simulink model for Sliding Mode Controller using Pole Placement approach

In Sliding Mode Control (SMC) design, the first step is to parameterize the sliding surface such that the system constrained to the sliding surface exhibits desired system behavior. In SMC, once the system slides on the designed surface, the order of the system is reduced [3]. This can be demonstrated by treating the lumped uncertainties to be zero.

The pole placement problem has been a subject for research for a long time. It is well known that state feedback control is an efficient technique for the pole placement problem. For single-input systems, this problem is well understood but for multi-input systems, the pole placement problem is more complex.

Consider the following system presented by the state-space model;

$$\dot{X}(t) = AX(t) + BU(t) + W(X,t)$$

where the linear state feedback regulator control law defined as *U*= -*KX*, and *W(X,t)* is the nonlinear term and disturbance of the system.

For system in (2), the closed loop poles are eigenvalues of matrix (*A*-*BK*). To make the system more stable and has desired time response characteristics, few steps have to be considered. First, select desired pole locations, and then find the gain matrix K which places the closed-loop poles to the desired locations.

Under state feedback *u*= - *Kx*, the closed-loop dynamics are given by:

$$\dot{x} = (A - BK)\, x$$

The closed-loop poles are the eigenvalues of (*A*-*BK)*. Using the '`place`' function in MATLAB, a gain matrix K can be computed, that assigns these poles to any desired locations in the complex plane (provided that *(A, B)* is controllable).

State-Feedback Gain Selection for finding Sliding Matrix S

For the Electrohydraulic control system model as described in (2), the pole placement method has been used get the gain matrix K. Using MATLAB, the value of S_1 can be found.

Firstly, the system matrices; A, B and C are entered. Then the poles of the system are determined using MATLAB function '*eig(A)*'. Three poles are obtained;

p1 = -149.9820
p2= 0
p3= -19.9982

One of the poles is at origin, which means that the system is marginally stable in open-loop transfer function. With a nonzero initial condition of x(0)= [0.005 0 0], the system response is shown in Figure 3.

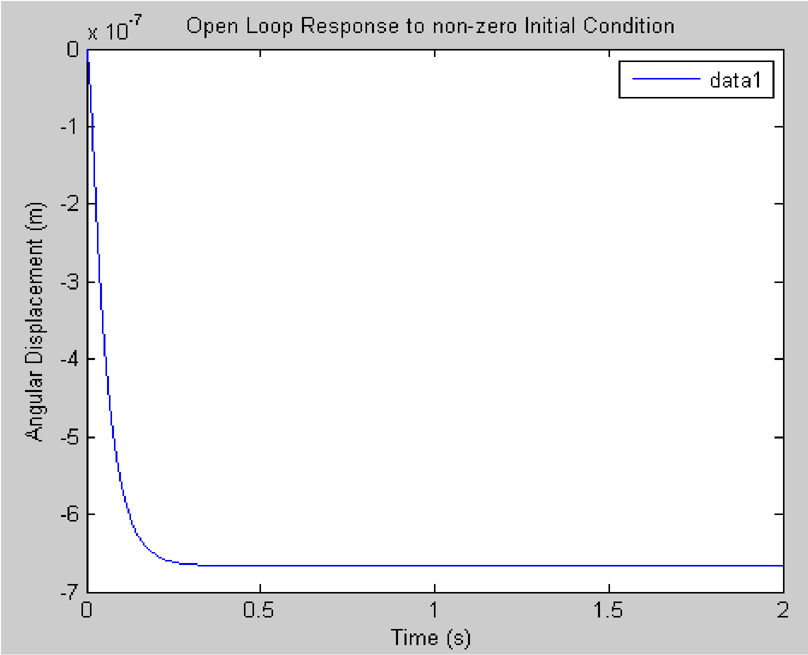

Figure 3: System response with a nonzero initial condition

From Figure 3, it can be seen that the system is stable but the transient response could be improved by selecting appropriate poles. The next step is to make the system more stable, poles should be adjusted, placed further to the left; this will improve the transient response and make the response to be faster. This time the overshoot is smaller. Then, the control matrix K is found using the Matlab function '*place*', which will give the desired poles. For a settling time < 0.5 sec and overshoot < 5%, the two dominant poles are placed at -10 +/- 10i. The third pole is placed at -50.

$$p1 = -10 + 10i$$
$$p2 = -10 - 10i$$
$$p3 = -50$$

Using the new poles values, the resulting response is shown in Figure 4 below.

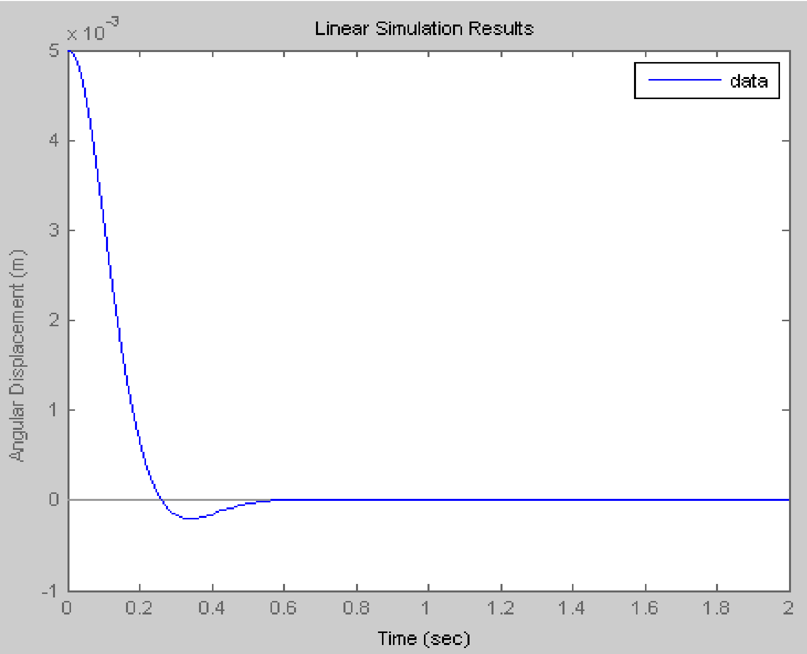

Figure 4: System response with the new poles values.

As seen in the Figure 4, the stability of the response has been improved. The obtained gain matrix K is;

K = [-0.1171 0.0211 0.0012]

Next, from the K value, the Sliding Matrix S is found. The sliding matrix can be obtained from:

$$\frac{S_1}{S_2} = K$$

By assuming $S_2 = I$, then S_1 = [-0.1171 0.0211 0.0012]

Lastly, this sliding matrix, S is substituted into the control law below [1], and

$$u(t) = -(SB)^{-1}K\sigma - (SB)^{-1}SAx - \rho(\omega,t)\frac{B^T S^T \sigma}{\|B^T S^T \sigma\|}$$

apply it to the Electrohydraulic Servo system plant. The resulting responses are observed for the position and velocity of the motor shaft as shown in Figure 5.

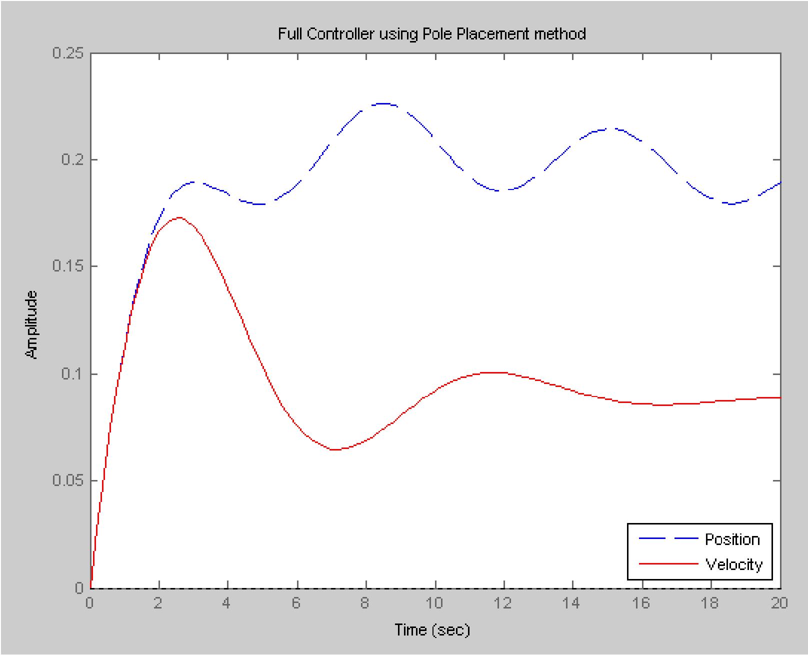

Figure 5: Angular Position and Velocity for Full Controller using Pole Placement method

The results of the angular position of the motor shaft responses using Pole Placement method and by using different controller (Sliding Mode and State Feedback Controller) are shown in Figure 6.

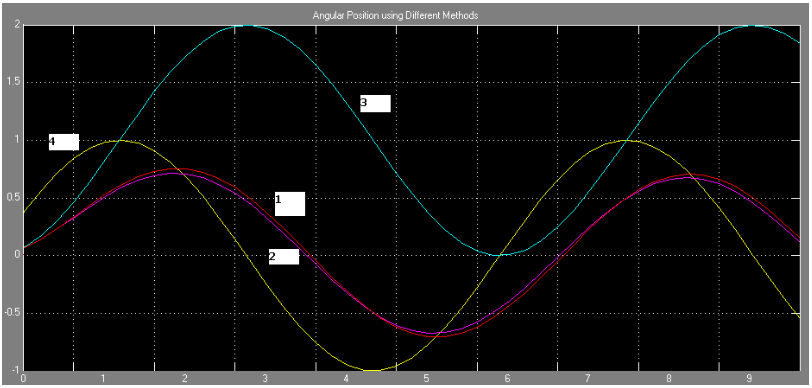

Figure 6: Angular Position Comparison using Different Methods

The different waveforms in Figure 6 are as follows:

1- Desired
2- SMC using LMI approach
3- State Feedback controller using Pole Placement approach
4- SMC using Pole Placement approach

Conclusion

Sliding Mode Control design involves two crucial steps; the first phase is to design a set of sliding manifolds so that the system state restricted to them have desired dynamics. The second phase is to design switching feedback control so that the system state trajectories can be attracted to the designed sliding manifold in finite time and maintained on the manifold. By applying the proposed controller, the perturbed sliding surface equation is enforced to zero and by an appropriate choice of this surface, the tracking error tends asymptotically to zero in finite time and with no chattering problems. In this paper, it is showed that the sliding mode controller designed based on the pole placement approach performed better than the full state feedback controller (without sliding mode) designed based on pole placement approach.

Acknowledgment

The author would like to thank Assoc. Prof. Dr. Mohamad Noh Ahmad from UTM Johor, for introducing a method of non-linear control system, sliding mode control which is involved in this project.

References

[1] Choi, Han Ho. An Explicit Formula of Linear Sliding Surfaces for a Class of Uncertain Dynamic Systems for a Class of Uncertain Dynamic Systems with Mismatched Uncertainties. Automatica, Vol. 34, No. 8, pp. 1015-1020, 1998.
[2] Fung, Rong-Fong, Yang R-T. Application of VSC in Position Control of a Nonlinear Electrohydraulic Servo System. Pergamon,Vol 66, No. 4, pp. 365-372, 1997.
[3] Spurgeon, S.K. and Edwards, C. Sliding Mode Control: Theory and Applications. Taylor and Francis, London, 1998.
[4] Merrit, Herbert E. Hydraulic Control System. Wiley, New York, 1976.
[5] Ha, Q. P., Nguyen, H. Q. and Durrant-Whyte, H.F. Sliding Mode Control with Fuzzy Tuning for an Electrohydraulic Position Servo

System. 2nd International Conference on Knowledge-Based Intelligent Electronic Systems, April 1998.

[6] Edwards, C., Spurgeon, S.K. Linear Matrix Inequality Methods for Designing Sliding Mode Output Feedback Controllers. IEE Proceedings- Control Theory Applications, Vol. 150, No. 5, September 2003.

[7] Hussain, A.R., Ahmad, M.N., and Mohd. Yatim, A.H. Sliding Mode Control with Linear Quadratic Hyperplane Design: An application to an Active Magnetic Bearing System. The 5th Student Conference on Research and Development-SCOReD 2007, December 2007.

[8] Decarlo, A. Raymond, Zak, H. Stanislaw, & Matthews, P. Gregory. Variable Structure Control of Nonlinear Multivariable Systems: A Tutorial. IEEE Transactions- Automatic Contro, Vol. 76, No. 3, March 1998.

[9] Chen, Chi-Tsong. Analog and Digital Control System Design: Transfer Function, State-Space, and Algebraic Methods. Saunders College Publishing.

[10] Hwang, C.L. Sliding Mode Control using Time-varying Switching Gain and Boundary Layer for Electrohydraulic Position and Differential Pressure Control. IEE Proc.-Control Theory Appl, Vol. 143, No. 4, July 1996.

[11] Dorf, R.C. Modern Control Systems. New York: Addison-Wesley, 1992.

[12] A. Decarlo, Raymond, H. Zak, Stanislaw & P. Matthews, Gregory. Variable Structure Control of Nonlinear Multivariable Systems: A Tutorial. IEEE Proceedings, Vol. 76, No. 3, March 1998.