

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

HANKEL DETERMINANTS FOR CERTAIN CLASS OF
ANALYTIC FUNCTIONS SUBORDINATE WITH
EXPONENTIAL FUNCTIONS

P19M19

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

In geometric functions theory, there are many subclasses that had been identified and introduced by researchers. It is normal for researchers to introduce new subclass of functions and find the related properties of that class of functions. We found out that there are not many existing projects that study on Hankel determinant for certain class of analytic functions subordinate with exponential functions. In this project, we aim to investigate the Hankel determinants which are Fekete-Szegö functional, second Hankel determinant and third Hankel determinant for the class of functions subordinate with exponential functions. So, we introduced the new subclass of analytic functions subordinate to exponential functions, K_α^* which satisfy

$$\frac{zf'(z) + \alpha z^2 f''(z)}{f(z)} \prec e^{w(z)}$$

where $0 \leq \alpha \leq 1$ and $z \in \mathbb{H}$. A few methods are used to get the results for the Hankel determinants such as comparison of coefficient, triangle inequalities, application of lemmas and maximization of functions. In conclusion, we had found the Hankel determinants which are Fekete-Szegö functional, second Hankel determinant and third Hankel determinant of our new subclass of function subordinate with exponential function in this project. The findings of this study will give contribution to geometric function theory where the new class of analytic functions is defined. The result obtained from this research will be applicable to the new researcher to develop new class of functions and identified the bounds on Hankel determinants. So, we hope that other researchers in the future research can introduce some new defined subclasses of analytic functions subordinate to exponential functions and its properties.