A Channel Matrix Rank Reduction Method Using Space – Time Coding in a MIMO System

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Abstract: Massive MIMO has an important role in 5G and IoT, however provides hardware problems in the system. To reduce these challenges we introduce a method for channel matrix rank reduction which reduces the computations volume in a $m{\times}n$ MIMO system using space – time coding in this paper. We used this method to $4{\times}2$ system to change its rank to $2{\times}2$, however it may be used to any desirable higher order $m{\times}n$ Massive MIMO systems.

Keyword: Matrix, Rank, Massive MIMO, Wireless.

1. Introduction

Massive MIMO incorporate reliability, high throughput and reducing the energy reduction and the interference to the wireless communication networks which are very applicable in 5G and IoT [1].
Symbols are affected from the multipath fading in wireless channels. To overcome to this challenge multi-antennas has been used in wireless systems [2, 3, 4]. Alamouti presented a simple and efficient algorithm in 1998 to eliminate the fading in multipath channels [5].
The Alamouti's presented method has provided a property through time diversity that the receiver needn't to the channel information and it can extract the channel coefficients from the received signals. After Alamouti, many different contributions based on precoding and antennas properties are presented to enhance the bit rate [6, 7, 8]. We intend to simplify the Alamouti algorithm in this paper. In [9] authors proposed a full-rate full-diversity STBC for 2×2 (MIMO) systems with a lower maximum likelihood (ML) detection complexity than that of existing schemes.

We offer a method to reduce the channel matrix rank and the volume of the computations by transforming the space diversity to the time diversity in the transmit side in this paper. We use the result to a $4{\times}2$ system to transform it to a $2{\times}2$ Alamouti coding. Thus, in section 2 we study the space-time diversity signals. In section 3 we study the transformation effects on transforming $4{\times}2$ to $2{\times}2$ system using the STC on the channel matrix and finally we describe the conclusion.

2. The system model

Fig. 1 depicts a $m{\times}n$ MIMO system. s_1, $s_2, \ldots, s_m$ signals have been transmitted simultaneously from m antennas in the transmitter and r_1, $r_2, \ldots, r_n$ signals have been simultaneously received by n antennas in the receiver. We have only considered the space diversity in this figure.
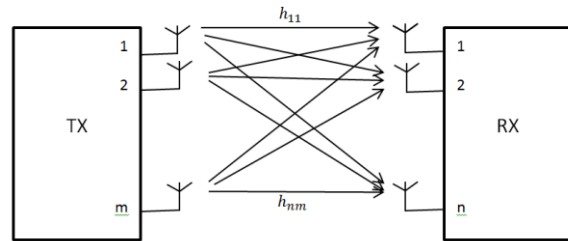Channel matrix H is of $m{\times}n$ order and here, we have been spread it into two parts as is shown in (1).

Fig.1 A $m{\times}n$ MIMO system

$$\begin{pmatrix} r_1 \\ \cdot \\ r_n \end{pmatrix} = \begin{pmatrix} h_{11} & \cdots & h_{1m/2} & h_{1,\frac{m}{2}+1} & \cdots & h_{1m} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm/2} & h_{n,\frac{m}{2}+1} & \cdots & h_{nm} \end{pmatrix} \begin{pmatrix} s_1 \\ \cdot \\ s_m \end{pmatrix}$$

(1)

To define our algorithm we divide the input signals into two sets $s_1, s_2, \ldots, s_{m/2}$ and $s_{m+1/2}$, $\ldots, s_m$ but transmit them during two consecutive T_1 and T_2 durations. In addition, we divide the number of transmitted antennas by 2. We call this a two order time diversity system. We know that to detect the transmitted symbols we should find

the inverse of the channel matrix H. Thus, the received signals are according to Table 1.

Table 1 Order two time diversity

T_2	T_1
r_{12}	r_{11}
r_{22}	r_{21}
.	.
.	.
.	.
r_{n2}	r_{n1}

Right subscript n in r_{mn} refers to the number of duration and left subscript m refers to the serial number of antennas. Besides, the right column in the table shows the received signals in the first duration and the left column shows the received signals in the second duration. Here we find two sets of equations with $m/2$ equations each. We assumed the channel matrix doesn't change during the T_1 and T_2.

$$\begin{pmatrix} r_{11} \\ . \\ r_{n1} \end{pmatrix} = \begin{pmatrix} h_{11} & \cdots & h_{1m/2} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm/2} \end{pmatrix} \begin{pmatrix} s_1 \\ . \\ s_{m/2} \end{pmatrix} \quad (2)$$

and

$$\begin{pmatrix} r_{12} \\ . \\ r_{n2} \end{pmatrix} = \begin{pmatrix} h_{11} & \cdots & h_{1m/2} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm/2} \end{pmatrix} \begin{pmatrix} s_{\frac{m}{2}+1} \\ . \\ s_m \end{pmatrix} \quad (3)$$

We sum the receive vector on the left of (2) and (3) and concatenate the matrices on the right of (2) and (3) to constitute (1) again, but here we reach to (4).

As it is shown, summing the received signals through two consecutive T_1, T_2 times evolves the channel matrix H in (1) to the channel matrix in (4).

$$\begin{pmatrix} r_{11}+r_{12} \\ . & . \\ r_{n1}+r_{n2} \end{pmatrix} = \begin{pmatrix} h_{11} & \cdots & h_{1m/2} h_{11} & \cdots & h_{1m/2} \\ \vdots & \ddots & \vdots \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm/2} h_{n1} & \cdots & h_{nm/2} \end{pmatrix} \begin{pmatrix} s_1 \\ . \\ s_m \end{pmatrix} \quad (4)$$

We can continue and divide both the number of transmitted signals and the input signals into three sets, etc. We have used here, order 3 diversity in the transmitter. Thus, the received signals change as indicated in Table 2 in which

the right, middle and left columns correspond to first, second and third received signals respectively. Besides, every row is pertinent to the corresponding transmitted signal in each antenna.

Table 2 Order three time diversity

T_3	T_2	T_1
r_{13}	r_{12}	r_{11}
r_{23}	r_{22}	r_{21}
.	.	.
.	.	.
.	.	.
r_{n3}	r_{n2}	r_{n1}

Therefore the input/output relation reveals to

$$\begin{pmatrix} r_{11}+r_{12}+r_{13} \\ . \\ r_{n1}+r_{n2}+r_{n3} \end{pmatrix} = $$
$$\begin{pmatrix} h_{11} & \cdots & h_{1m/3} h_{11} & \cdots & h_{1,2m/3} h_{11} & \cdots & h_{1m/3} \\ \vdots & \ddots & \vdots \vdots & \ddots & \vdots \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm/3} h_{n1} & \cdots & h_{n,2m/3} h_{n1} & \cdots & h_{nm/3} \end{pmatrix} \begin{pmatrix} s_1 \\ . \\ s_m \end{pmatrix} \quad (5)$$

As it is shown, summing the received signals in the first, second and third durations on each antenna divides the H matrix to three equal parts and reveals the equation to (3). We can extend the above procedure to provide a p order time diversity in $T_1, T_2, \ldots, T_p$ times as:

$$\begin{pmatrix} \sum_{i=1}^{p} r_{1i} \\ \ldots \\ \sum_{i=1}^{p} r_{ni} \end{pmatrix} = $$
$$\begin{pmatrix} h_{11} & \cdots & h_{1m/p} & h_{11} & \cdots & h_{1m/p} \\ \vdots & \ddots & \vdots & \cdot & \cdot & \cdot & \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm/p} & h_{n1} & \cdots & h_{nm/p} \end{pmatrix} \begin{pmatrix} s_1 \\ . \\ s_m \end{pmatrix} \quad (6)$$

Assuming

$$H_p = \begin{pmatrix} h_{11} & \cdots & h_{1m/p} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm/p} \end{pmatrix} \quad (7)$$

we may rewrite (6) as

$$\begin{pmatrix} \sum_{i=1}^{p} r_{1i} \\ \ldots \\ \sum_{i=1}^{p} r_{ni} \end{pmatrix} = \begin{pmatrix} H_p & \cdot & \cdot & \cdot & H_p \end{pmatrix} \begin{pmatrix} s_1 \\ . \\ s_m \end{pmatrix} \quad (8)$$

which means that H is

$$H = \begin{pmatrix} H_p & \cdot & \cdot & \cdot & H_p \end{pmatrix} \quad (9)$$

As it is shown, the matrix H includes p identical H_p matrix which its row is equal to the number of received antennas and the number of columns is equal to the transmitted antennas $(n \times p)$, i.e., the number of columns has changed inversely to diversity order p. Besides, the number of H_p matrices in (9) is equal to p time diversity. We have indicated in Appendix that we can find the inverse of H easily as

$$H^{-1} = \left(\frac{H_p^{-1}}{p} \cdot \quad \cdot \quad \frac{H_p^{-1}}{p} \right)^T \qquad (10)$$

where T in (10) is the transpose indication. Thus, using the p time transmit diversity, to get the inverse of a large channel H matrix, we need only find the inverse of H_p matrix and then by repeating it in a matrix as in (10) we will find the inverse of that large H matrix. When the number of columns and rows are not equal, we may find pseudo inverse of the matrix H according to the above procedure, i.e.

$$H^+ = \left(\frac{H_p^+}{p} \cdot \quad \cdot \quad \frac{H_p^+}{p} \right)^T \qquad (11)$$

3. Verifying and Evaluation

We assumed implicitly that the channel characteristics doesn't change in successive durations. To verify this assumption we should indicate that $T_s \ll T_c$ in which T_s is the symbol period and T_c is the channel coherence time. As we know e.g. in LTE-A BW=100Mbps or the symbol period is $T_s = 100ns$. On the other hand $T_c \approx \frac{1}{f_d}$ in which f_d is Doppler spread. Now let the vehicle speed is $v_h = 200 km/h$ and the carrier frequency is $f_c = 2\text{GHz}$, furthermore $f_d = 100Hz$ or $T_c \approx 10ms$. Thus $10ns \ll 10ms$ which confirms that we may use our method [10].

For evaluation the method we should indicate the simplicity in the computations. We study a 4×2 MIMO system which has the form

$$\begin{pmatrix} r_1 \\ \\ r_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} \qquad (12)$$

Thus, the inverse of channel matrix is

$$H^+ = H^h \left(HH^h \right)^{-1} \qquad (13)$$

But, using the 2×2 MIMO and order 2 diversity, (12) will change to

$$\begin{pmatrix} r_1 \\ \\ r_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{11} & h_{12} \\ h_{21} & h_{22} & h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} \qquad (14)$$

in which

$$\begin{cases} r_1 = r_{11} + r_{12} \\ r_2 = r_{21} + r_{22} \end{cases} \qquad (15)$$

Channel matrix H in (14) has been constituted from two 2×2 identical matrices which we may find its inverse from (10). Let

$$H_2 = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \qquad (16)$$

then easily we find

$$H_2^{-1} = \frac{1}{h_{11}h_{22} - h_{21}h_{12}} \begin{pmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{pmatrix} \qquad (17)$$

and consequently,

$$H^+ = \left(\frac{H_2^{-1}}{2} \middle| \frac{H_2^{-1}}{2} \right)^T =$$
$$\frac{1}{2(h_{11}h_{22} - h_{21}h_{12})} \begin{pmatrix} h_{22} & -h_{12} & h_{22} & -h_{12} \\ -h_{21} & h_{11} & -h_{21} & h_{11} \end{pmatrix}^T \qquad (18)$$

and therefore

$$\begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \frac{1}{2(h_{11}h_{22} - h_{21}h_{12})} \begin{pmatrix} h_{22} & -h_{21} \\ -h_{12} & h_{11} \\ h_{22} & -h_{21} \\ -h_{12} & h_{11} \end{pmatrix} \begin{pmatrix} r_1 \\ \\ r_2 \end{pmatrix} \qquad (19)$$

To calculate the volume of the calculations in two methods we see that to find H$^+$ in from (13) we need to perform 3 multiplications and a matrix inversion, but in the second method we need only one matrix inversion from (17) and duplicate it in a new matrix according to (18). Thus, we find that calculating H^{-1} from (19) is much easier than calculating it from H in (13).

4. Conclusion

Using the space-time coding procedure and the aforementioned method we simplified the channel matrix and the computations. Since we transform the higher order channel matrices to

lower orders matrices, this simplification reveals better in higher orders of channel matrices.

Appendix

We prove (10) as a statement in the following paragraph.

To multiply and making inverse of the channel matrix H, we use the dividing procedure in the matrices. Assuming $H_{n\times m}$ and $B_{m\times n}$ are inverse, thus HB=I. To provide B we divide H and B in e.g. 3 sub-matrix somehow that their multiplication are possible as

$$(H1_{n\times q} \quad H2_{n\times q} \quad H3_{n\times q}) \begin{pmatrix} B1_{q\times n} \\ B2_{q\times n} \\ B3_{q\times n} \end{pmatrix} = I_{n\times n} \qquad \text{(a)}$$

in which I is a unitary matrix. Thus we find

$$H1.B1+H2.B2+H3.B3=I_{n\times n} \qquad \text{(b)}$$

and therefore

$$\begin{cases} H1.B1 = \frac{1}{p}I_{n\times n} \\ H2.B2 = \frac{1}{p}I_{n\times n} \\ H3.B3 = \frac{1}{p}I_{n\times n} \end{cases} \qquad \text{(c)}$$

in which $p=\frac{1}{3}$ and thus

$$B1=\frac{1}{p}H1^{-1}, \qquad B2=\frac{1}{p}H2^{-1}, \qquad B3=\frac{1}{p}H3^{-1}$$
(d)

Incorporating the equations (d) in a matrix form, we get H^{-1} as is indicated in (10).

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