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EFFECTIVENESS OF VISUAL REPRESENTATIONS IN PROMOTING EXACT SELECTION OF CONTINUITY CORRECTION

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Abstract

The Square + Line method was formulated to enhance the understanding of students in cases involving Normal Approximation to either Binomial Distribution or Poisson Distribution. In these topics, usage of continuity correction factor is essential, specifically when we apply continuous function to approximate a discrete one. The simple steps of modifying (adding or subtracting 0.5) the discrete x-value are made complicated by the various interval situations. Through this approach, students can visualize the correct conversion for all the nine transformation cases. While it is possible that this method does not benefit quick learners, it nevertheless helps students who have difficulty with continuity knowledge retention. To measure the effectiveness of this method, students who took diploma program in computer science were given pre- and post-assessment before and after the learning session. The comparatively much higher post-test score indicates a significant improvement.

Keywords*: Visualization; Normal approximation; Continuity correction; Paired t-test*

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1. Introduction

In mathematical study, problem solving has been an essential element that greatly influences the learning outcomes of students. Learning how to solve mathematical problems is crucial in order to achieve satisfactory results. Ibrahim's study (as cited in Tambychik & Meerah, 2010) found that mathematics problem solving is not a topic but a process that underlies the whole mathematics programmes which contextually helped concepts and skills to be learned. In line with this, students need to have the right concept, method and skills for any mathematical solution. However, every back and then, they seem to have difficulties in adopting the correct problem solving approach. This is due to the inability to acquire the basic skills they need in mathematics (Phonapichat, Wongwanich, & Sujiva, 2014; Tambychik & Meerah, 2010). Nevertheless, this issue can be solved by adding visualization techniques into the learning atmosphere of mathematics classroom, in which it combines dexterous presentation of drawing and problem solving (Kashefi, Alias, Kahar, Buhari, & Mirzaei, 2015). Mathematics teaching and learning needs to become more visual as there is not a single idea or concept that cannot be illustrated or thought about visually (Boaler, Chen, Williams, & Cordero, 2016). Therefore, visualization plays an important role in teaching and learning mathematics.

1.1 Role of visualization in mathematics problem solving

Visualization as an epistemological learning tool in mathematics, thus, begins with this strong assumption in which case the minds of learners are not seen as unaided or unconstrained but committed to look in particular ways that support the emergence of necessary mathematical knowledge (Rivera, Steinbring, & Arcavi, 2014). As such, visualization can be a powerful tool to explore mathematical problems and to give meaning to mathematical concepts and the relationship between them (Rosken & Rolka, 2006). Through visual interpretations, students are able to conceptualize ideas and have better understanding of what they have learned (Caligaris, Rodríguez, & Laugero, 2015). Having all of these definitions and statements, visualization cannot be isolated from the rest of mathematics, where symbolic, visual and numerical representations must be connected (Caligaris et al., 2015). We are convinced that visualization is indeed a valuable alternative to assists learners or students in comprehending numerous topics in mathematics.

1.2 Visualization in correction for continuity factor

As visualization offers various solution methods to mathematics, it can also be applied in correction continuity factor. A correction for continuity factor is applied when a continuous function is used to approximate a discrete function. However, in reducing the approximation error, Yates (as cited in Stefanescu, Berger, & Hershberger, 2005) suggested a correction for continuity that adjusts the formula for Pearson's chi-squared test by subtracting 0.5 from the difference between each observed value and its expected value in a 2×2 contingency table. Mastering the concept of when to add or subtract 0.5 factor is like climbing a very steep stepladder. While first few steps are easily accomplished without much mental distraction, users tend to hesitate when reaching the middle level. At this stage, fear of falling and safety worries will start to appear, disrupting the climbing sequence. To clear the air, let us consider the following problems: $P(x < a)$ and $P(x <= a)$. Their conditions are quite similar to each other but different solutions are opted instead. In general, students can solve direct cases such as $P(x > a)$ and $P(x < a)$. However, confusion arises when number of cases is diversified to $P(x <= a)$ or $P(x >= a)$, as in the event of mid-level ladder. Many of them would perceive the ($x < a$, $x <= a$) or ($x > a$, $x >= a$) inequalities to be identical, thereby applying the same solution. In teaching the above concept, a visualization on correction for continuity factor was developed, using appropriate symbols. The main purpose of this visual method is that learners or students are able to distinguish the correct adjustment with the corresponding method without any hesitation. Through visualization, they will develop better understanding on how to deal with mathematical situations and problems. This chronology of methods can also promote a different kind of mathematical thinking (Caligaris et al., 2015).

2. Method

2.1 Participants and materials

Students taking the Introduction to Probability and Statistics course (Diploma in Computer Science, UiTM Sarawak) were enrolled in this study. They studied the Normal Approximation to Binomial Distribution topic in Chapter 4. We adopted the quasi experimental design for our case study, where the use of pre- and post-test were exclusively distributed to a solitary group. To complement the absence of control group, a mixed method questionnaire, consisting of six preceded rating scale and four open-ended free text questions, was incorporated towards the end of post-assessment.

Each of the pre- and post-assessment contains ten continuity correction transformation problems. These questions (see Appendix) were carefully designed to encompass various interval situations. While a similar set of problems were used, the order of questions within pre- and post-tryout has been reshuffled. As for the questionnaire (see Appendix), Q1-Q4 referred to the overall impression of participants about basic statistics. Q5-Q6 requested participants to opine their views on the Square + Line method. The final four questions encouraged participants to write out their thoughts and suggestions regarding this method, without any prefabricated responses.

2.2 Introducing continuity correction factor

Applying and selecting appropriate continuity correction factor is key to solving problems related to aforementioned topic. Conventionally, students are taught to solve related problems by referring to the following table:

$$\text{a) } P(X \geq a) \xrightarrow{cc} P(X > a - 0.5)$$

$$\text{b) } P(X > a) \xrightarrow{cc} P(X > a + 0.5)$$

$$\text{c) } P(X \leq a) \xrightarrow{cc} P(X < a + 0.5)$$

$$\text{d) } P(X < a) \xrightarrow{cc} P(X < a - 0.5)$$

$$\text{e) } P(a \leq X \leq b) \xrightarrow{cc} P(a - 0.5 < X < b + 0.5)$$

$$\text{f) } P(a < X < b) \xrightarrow{cc} P(a + 0.5 < X < b - 0.5)$$

$$\text{g) } P(X = a) \xrightarrow{cc} P(a - 0.5 < X < a + 0.5)$$

2.3 Introducing the Square + Line method

This approach encourages students to illustrate the mathematical statement, and along the line becoming more adept in the visualization and manipulation of discrete to continuous conversion procedure. The innovative method was only imparted to students after they sat the pre-test. While learning this visualization method, students were merely required to relate prior inequality and interval knowledge. **Figure 1** illustrates the rules applied when assimilating this method. Step-by-step application of these rules is shown in **Table 1-6**.

≤ or ≥ means square included, therefore the square is filled

< or > means square excluded, therefore the unfilled square is used

= means only consider the square, no line is drawn

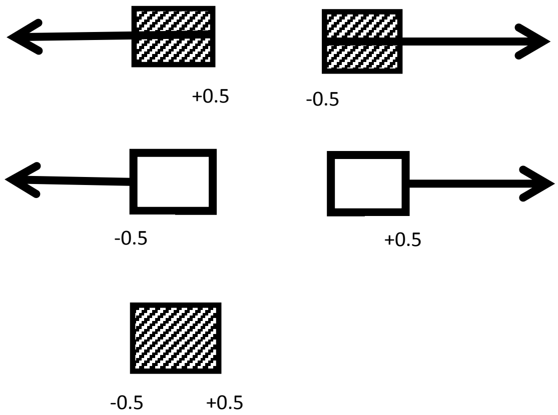


Figure 1 Square + Line rules

Table 1 Write down the transformation for P(X ≥ 21)

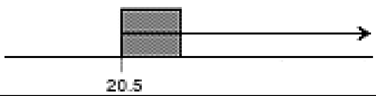
Step 1	Draw the number line.	
Step 2	Draw a square of number 21 and an arrow to the right.	
Step 3	We can now write the transformation.	$P(X \geq 21) \xrightarrow{cc} P(X > 20.5)$

Table 2 Write down the transformation for P(X < 44)

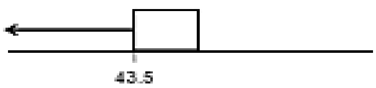
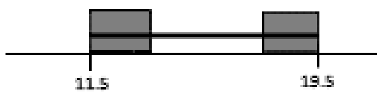
Step 1	Draw the number line.	
Step 2	Draw the square of number 44 and arrow to the left.	
Step 3	We can now write the transformation.	$P(X < 44) \xrightarrow{cc} P(X < 43.5)$

Table 3 Write down the transformation for P(12 ≤ X ≤ 19)

Step 1	Draw the number line.	
Step 2	Draw the square of number 12 and 19 and the line between the two squares.	

Step 3	We can now write the transformation.	$P(12 \le X \le 19) \xrightarrow{cc} P(11.5 < X < 19.5)$

Table 4 Write down the transformation for P(66 ≤ X < 73)

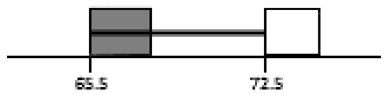
Step 1	Draw the number line.	
Step 2	Draw the square of number 66 and 73 and the line between the two squares.	
Step 3	We can now write the transformation.	$P(66 \le X < 73) \xrightarrow{cc} P(65.5 < X < 72.5)$

Table 5 Write down the transformation for P(48 < X < 54)

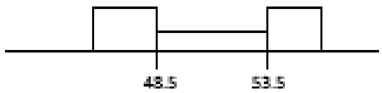
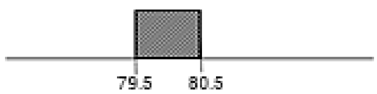
Step 1	Draw the number line.	
Step 2	Draw the square of number 48 and 54 and the line between the two squares.	
Step 3	We can now write the transformation.	$P(48 < X < 54) \xrightarrow{cc} P(48.5 < X < 53.5)$

Table 6 Write down the transformation for P(X = 80)

Step 1	Draw the number line.	
Step 2	Draw the square of number 80.	
Step 3	We can now write the transformation.	$P(X = 80) \xrightarrow{cc} P(79.5 < X < 80.5)$

4. Findings and Discussion

From the 21 sets of pre- and post-assessment handed out, only 16 sets are considered for analysis (n=16). The remaining five who completed either one of the pre- or post-assessment are excluded.

Table 7 Score attained by participants in pre- and post-test

No	Student	Pre-assessment	Post-assessment	Difference
1	S1	10	10	0
2	S2	7	10	3
3	S3	9	8	-1
4	S4	10	10	0
5	S5	7	10	3
6	S6	8	10	2
7	S7	3	9	6
8	S8	9	10	1
9	S9	9	9	0
10	S10	5	10	5
11	S11	10	8	-2
12	S12	10	10	0
13	S13	10	9	-1
14	S14	2	2	0
15	S15	4	10	6
16	S16	2	10	8

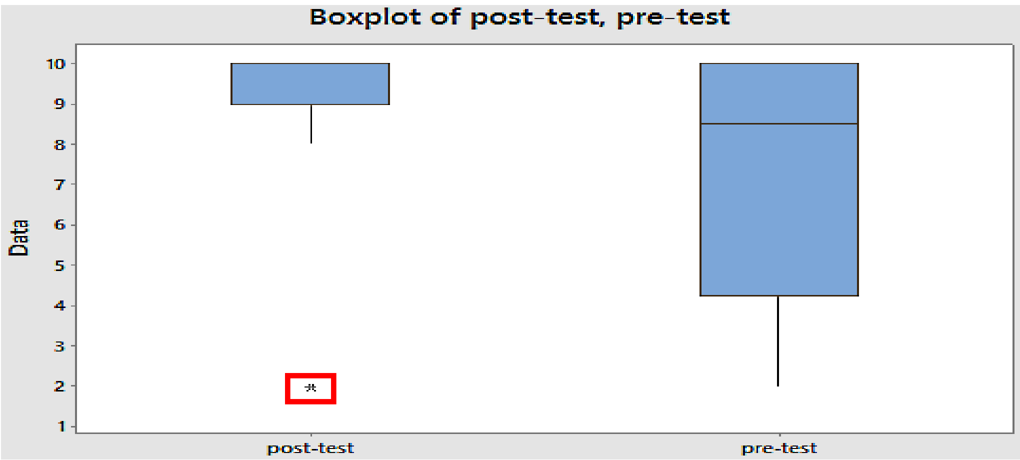
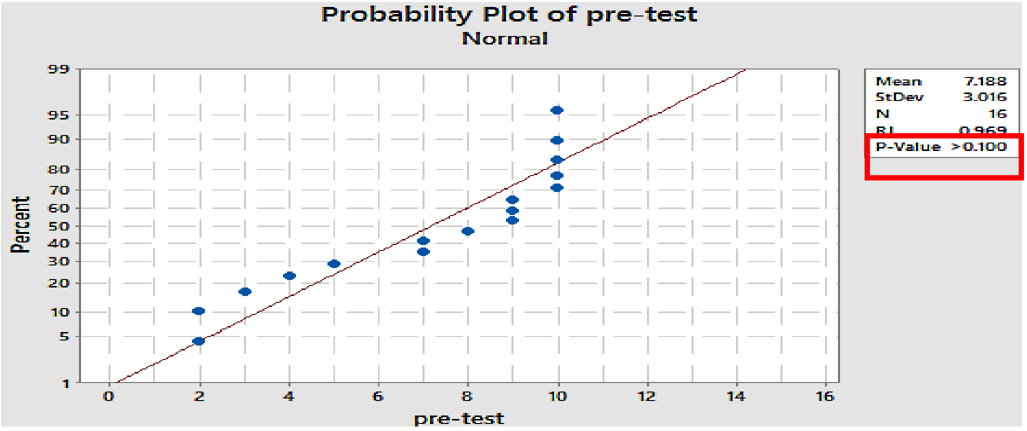


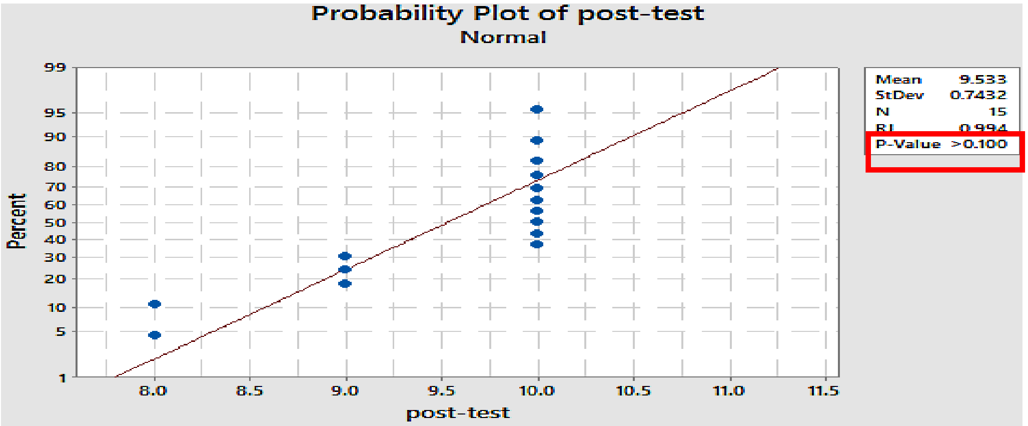
Figure 2 Boxplot

Boxplot of these data (**Figure 2**) suggests appearance of an outlier (*) in post- dataset. Upon removal of the sole outlying value, both datasets when analyzed via Ryan-Joiner test portray normal distribution, as highlighted by **Figure 3**.

As can be seen in **Table 8**, participants obtain 7.53 and 9.53 scores in pre- and post-assessment, respectively. These results (t = 2.54, $p < 0.05$) indicate comparatively better performance in the post assessment, thereby signifying the effectiveness of our visualization tool in improving learners' comprehension of continuity correction.



(a)



(b)

Figure 3 Normal probability plot for (a) pre- and (b) post- data

Table 8 Output from MINITAB

Paired T-Test and CI: post-test, pre-test

```
Paired T for post-test - pre-test

             N   Mean  StDev  SE Mean
post-test   15  9.533  0.743    0.192
pre-test    15  7.533  2.774    0.716
Difference  15  2.000  3.047    0.787


95% lower bound for mean difference: 0.614
T-Test of mean difference = 0 (vs > 0): T-Value = 2.54  P-Value = 0.012
```

A qualitative approach is also carried out to further look into the way the students answer the questions in their pre- and post-test. The focus is particularly on the students (S7, S10, S15 and S16) who have shown a difference of at least positive five in their pre- and post-test scores. In the pre-test of student S16 who scores two marks, three questions are answered in full, while no answer is given for the rest of the questions. The performance of S16 in pre-test suggests that s/he is having extremely little knowledge and skills in handling the continuity correction questions. Nevertheless, this student surprisingly managed to answer all of the questions correctly in the post-test where the diagram of Square + Line method is shown scribbled for all of the questions.

As for students S7 and S15, though all of the questions are answered in full, they only managed to score three and four marks respectively in the pre-test. The pre-test results suggest that they are confused and may experience misconception on whether to add or minus 0.5 in the continuity correction procedure. A closer look at the answers in the pre-test reveals that the students are particularly incompetent in answering questions related to cases involving “less than and equal to” scenarios. Student S7 and S15 obtained a score of nine and ten respectively for their post-test. Both of them used the Square + Line method for all of the questions by sketching them beside each of the questions. On the Square + Line diagrams sketched, they apparently write down a correct ‘+0.5’ or ‘–0.5’ on the correct position of either on the left or right of the filled or empty square of the Square + Line method. The question answered wrongly in the post-test of S7 involves the ‘equal’ scenario of the continuity correction questions, which is also wrongly answered in the pre-test. This has thus implied that S7 is yet to capture the concept of this one scenario, and further guidance could be provided to fill the knowledge gap.

On the other hand, student S10 who scores five marks in the pre-test shows that all the five questions that s/he has answered wrongly are those related to the ‘greater than’ scenario in the continuity correction even though s/he has answered all the questions on ‘greater than and equal to’ scenario correctly. It is thus obvious that student S10 possesses certain misunderstanding between these two scenarios. Full score obtained by S10 in post-test confirms that s/he has overcome his/her misconception. The use of Square + Line method by S10 in the post-test, shown by the diagram of the method sketched beside every question, implies that this method has assisted S10 in overcoming the above misconception.

Comparing the score of the students for pre- and post-tests, the number of students who have successfully scored full mark in the post-test (62.5 percent) is twice as many as those in the pre-test (31.25 percent). Those who score full mark in the pre-test have also performed impeccably in the post-test. Interestingly, student S4 who scores full mark in both of the pre- and post-test is noticed to draw a table representing his/her way of weighing whether to plus or minus 0.5 in the pre-test, later resorted to draw the Square + Line diagram beside each of the questions as the method to solve the questions in post-test. Apart from that, students S2, S5, S6 and S8 have got all the questions that they have answered wrongly in pre-test correct in the post-test, with the help of drawing the Square + Line in the post-test. Conversely, there is one student (S14) who scores two marks from the same questions in both of the pre- and post-test, suggesting no improvement in his/her knowledge pertaining to the topic of continuity correction. Though the diagram of Square + Line method is sketched in the post-test, some of them are used incorrectly. The student persistently faces problem especially with questions involving compound inequalities in both of the pre- and post-test. This phenomenon suggests that student S14 needs further help in understanding the concept and learning to visualize what is learned through the Square + Line method.

Nonetheless, there are a few results portrayed by a small number of students that have suggested possible improvement to be considered in teaching the above topics using the Square + Line method. Three students (S3, S11 and S13) who show a decrease in their scores in post-test are noticed to have mistakenly used or interpreted the Square + Line method, particularly in choosing either the filled or emptied square, in one or two of the questions. In the teaching and learning process of the above topics using the Square + Line method, special attention should be paid to ensure that the students are not confused with the representation of the filled and emptied square.

On top of the above analyses, the feedback collected from the questionnaire is also analyzed. When asked about whether the Square + Line method help, all the students responded positively. Some of them added that this method has made it easier to determine whether to add or minus 0.5, and with more practice in drawing the Square + Line method while solving the continuity correction questions, one is able to use the method without explicitly sketching them out on the paper. In responding to the question on the best thing about the method, the students comment that the Square + Line method has i) enabled them to visualize the rules in performing the continuity correction procedures; ii) made it possible for them to remember the procedure without memorizing the rules; iii) been not only very helpful and simple-to-use but is also an effective method; and iv) helped to enhance their understanding of the topics. All the students have responded negatively for the question asking for the worst thing of using the method; except that two of the students comment that the method could be confusing, which has been addressed above. For the question asking for improvement of the method, one student proposes the use of ready-made diagram tool for the method so that the act of drawing the method repetitively for every question could be avoided.

5. Conclusions

Study on the efficiency of Square + Line method has been conducted on students taking the Introduction to Probability and Statistics course. Analysis of their pre- and post-test performances points to a 26.1% increase in mean score, denoting significant post treatment improvement of their comprehension and self-efficacy levels. Moreover, qualitative investigation of their questionnaire feedback demonstrates a relatively small percentage of negative categorical rating. In general terms, this offers the impression that majority of participating students are satisfied with the product and have profited from it.

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Appendix 1

Pre-assessment

Name:	
Group:	
Student's ID:	

Write down the transformations for each of the following, when a normal distribution is to be used as an approximation for a binomial distribution / poisson distribution.

1. $P(4 \le X \le 9)$
2. $P(26 < X \le 28)$
3. $P(X > 19)$
4. $P(X \le 109)$
5. $P(X = 65)$
6. $P(54 \le X < 67)$
7. $P(x \ge 23)$
8. $P(X < 32)$
9. $P(5 < X < 13)$
10. $P(0 < X \le 10)$

Appendix 2

Post-assessment

Name:	
Group:	
Student's ID:	

Write down the transformations for each of the following, when a normal distribution is to be used as an approximation for a binomial distribution / poisson distribution.

	SQUARE + LINE
1. $P(X=65)$	
2. $P(0< X\leq 10)$	
3. $P(X>19)$	
4. $P(X\leq 109)$	
5. $P(5< X<13)$	
6. $P(54\leq X<67)$	
7. $P(x\geq 23)$	
8. $P(X<32)$	
9. $P(4\leq X\leq 9)$	
10. $P(26< X\leq 28)$	

Appendix 3

SQUARE + LINE

QUESTIONAIRE

Name : ______________________________

Gender : M/F

Course : ______________________________

Matric No. : ______________________________

Ethnic/Race: ______________________________

Please rate each of these statements on a scale of 1 to 5 (1 being lowest rating and 5 being highest rating).

		1	2	3	4	5
1	I like statistics in university.					
2	I use statistics outside of university.					
3	The statistics I learn at university is helpful.					
4	I am good at statistics.					
5	I liked the method of 'Square + Line'.					
6	The method of 'Square + Line' has enhanced my learning.					

7 Does the method 'Square + Line' help you?

____
____
____
____
____

8 The best thing about the 'Square + Line' is

____
____
____
____

9 The worst thing about the 'Square + Line' is

____
____
____
____

10 Do you have any suggestions on the improvement of this method?