

Solving River Blindness Disease in terms of Progression by using SIR

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ABSTRACT

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River blindness disease has become a major problem in Northern Nigeria, Africa especially in a rural agricultural land located to a nearby river edge. The population is at a higher risk of being infected for being very close to the breeding site of black fly or known as *Simulium damnosum*. The infected black fly caused the transmission of parasite to human body. This study focused on solving the River Blindness disease using the SIR model with vital dynamic. This paper aims to solve the ordinary differential equation (ODE) by using steady state theorem to identify the change percentage of susceptible, infected and recovery for the population in Northern Nigeria, Africa in a time period. The initial percentage of susceptible individual is 0.65 of the population, but only 0.01 of the susceptible individuals are infected with the disease and 0 of them has recovered. The progression of the disease after 90 days is predicted using the Euler Method in Matlab.

Keywords: ODE, SIR, Black Fly, and Euler Method.

1. INTRODUCTION

1.1 Background Research

River blindness is also known as Onchocerciasis. This disease is caused by parasite *Onchocerca volvulus* (worm) that was found deeper in tissues of a human body as shown in Figure 1.1 [17]. Females parasite release about 700-1500 microfilariae per day, and are mostly found in the skin, eyes and other tissues. The microfilariae was transmitted from the bite of the black fly (vector). This caused the microfilariae to develop into infective larvae and are transmitted again to a human body by the bite of the infectious black fly. According to a study by Hopkins and Boatman (2011) in about a year, the larvae develop into adult parasite and the cycle thus begins. Figure 1.2 shows the transmission of the disease by the parasite and also by humans [17].



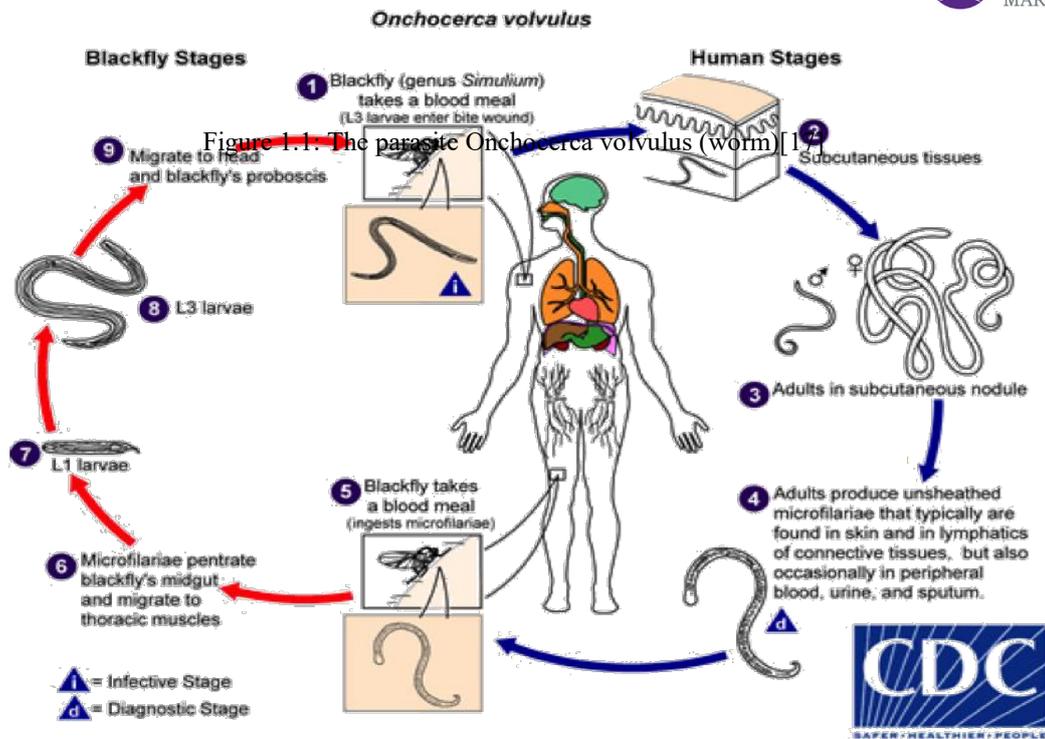


Figure 1.2 Transmission of the disease[17]

The microfilariae that have not been transmitted inside a human body start to die after 6 months. Thus the infected population with visible symptoms can be recognized. The infected population mostly have some of the conditioned skin disease or known as onchodermatitis. According to the research lead by World Health Organization. (2015) , the skin becomes itchy, hyperpigmented and start bleeding. Besides, microfilariae are also identified in the cornea of the eyes, thus leading to low visions and becoming white. According to Hopkins and Boatin (2011) the cornea condition may worsen and cause eye blindness for some infected population. The figures below show the skin disease and eye blindness which are suffered by the people who are infected with this disease[17].



Figure 1.3 Nodule



Figure 1.4 Onchodermatitis

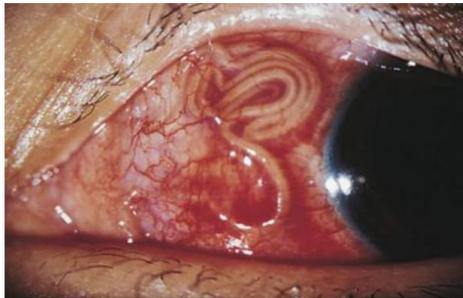


Figure 1.5 Microfilariae which migrate in the eye



Figure 1.6 Eye blindness [17]

River blindness disease is mostly extreme in Africa and the estimated number of population treated with Ivermectin from 1997 to 2005 were recorded by the World Health Organization (2015). Ivermectin is the mass administration of an oral drug being donated by Merck, considered able to increase the number of recovered population as shown in Figure 1.7 [4][9]. In addition, Figure 1.8 shows that the part of Onchocerciasis Control Programme (OCP), whereby helicopters and fixed wing craft are used to spread insecticides on rivers and river banks as these are the sites for the black fly breeding [3]. According to Sarwar (2016), insecticide types used are organophosphates, carbamates and pyrethroids. The elimination of black fly reproduction is also important as the number of flies increases, more black flies become infected when in contact with the infected individuals within the area. This will cause more cycles of transmission of disease between the black fly and humans.



Figure 1.7 Ivermectin



Figure 1.8 Control the breeding site of black fly

In general, any disease caused by parasites or other microorganisms can only be identified by behavior and transmission in detail through mathematical modeling. These epidemic models

have multiple compartments such as SI, SIS, SIR, SIRS, SEIR, SEIRS, SEI, SEIS and so on which in **S** stands for Suspected, **I**, infected, and **R** for Recovery. It is important to identify the characteristics of each disease before we choose the compartment to control and predict the disease in the future. There are various advancement situations for non-related disease exposure of bacteria. As an example, the infected population is likely to recover with immunity which can still be studied using the SIR model, while those recovering with temporary immunity will be studied using the SIRS model.

According to Trawicki (2016), the spreading disease involves exposure to bacteria, as the time taken to show the infected symptom is relatively long. Therefore, the SEIR model will be used to study the progression in detail for the change in the number in the class if recovered with permanent immunity while otherwise will be studied utilising the SEIRS model. In addition, according to Knipf & Rost (2011) to build a single disease model in one population. The population is divided into several classes such as group comprising the susceptible, exposed, infected and recovery. This makes studying the changes in population numbers in each class at a time easier.

Other factors that will cause a change in the number of groups include age, gender, social status and race. Generally, the low immunity condition among children, parents, chronic patients, pregnant women and other populations living close to the black fly breeding sites have a high percentage contributing to the increased rates of infection.

Based on Omade et al. (2015), the SIR model with vital dynamic (births and deaths) will be applied to compute the percentage of susceptible, infected and recovered individuals of the River blindness disease in Africa. Based on the nature of disease distribution in the population, the SIR model is most appropriate to be applied. Diekmann & Heesterbeek (2000) stated that through the SIR model, an individual who is entered into the system and eventually gave birth will be admitted into the susceptible compartment S as there will be a probable infection. $S(t)$ represents the number of susceptible individuals at a time. Then, the affected susceptible individuals will get into the infected compartment I , whereby $I(t)$ is the number of infected individuals at a time.

Finally, another study by Iannelli (2005) found that the infected population will enter the recovered compartment R , where this population is estimated to recover a long-life immunity. Hence, $R(t)$ is the number of recovered individuals at a time. So based on the concept of this model, the progression of the disease can be analyzed in more detail based on the change of number for the susceptible, infected and recovered population in Northern Nigeria, Africa over a period of time.

1.2 Problem Statement

According to Omade et al. (2015), the River blindness disease is most extreme in remote Africa (Northern Nigeria). Populations living along the edge of the river are at a higher risk compared to others that live further away. Thus, these population are very close to the breeding site of black fly or *Simulium damnosum*; the vector that caused the transmission of parasite to the human body.



Figure 1.10: An adult blackfly (*Simulium damnosum*) taking a blood meal on human skin[17]

Moreover, Omade et al. (2015) reported that the number of black fly population is assumed to be constantly bred/ to be in a constant breeding. In fact, the black fly in Africa has many complex species. The understanding for each species is important because it is related to the number of infectious black fly involved in transmission of disease by (Hopkins & Boatin, 2011). It is generally difficult to estimate the value of black fly because it is difficult to isolate and calculate the total species in a study area. For this project, the population of black flies is estimated to be equal or constant sizes. The significant impact of River Blindness disease for the population in Northern Nigeria, Africa has led to the establishment of a new method of control.

A mathematical analysis, in the form of the SIR model with vital dynamic (births and deaths) will be applied for the River blindness victims to achieve a further understanding of the progression disease especially where the changes of number for susceptible, infected and recovered in the population are concerned. The limitations of the SIR model in this project do not take into account other factors that affect the development of disease in the population. According to Remme (2004), there are differences in age and gender for population infected with the River Blindness disease. Every infected population shows different symptoms such as blindness, low vision and itching. Therefore, in order to control the disease in the population, such factors should be emphasized in order to focus more on a specific age and gender group.

The next research considered the SIR model and derivation of Ordinary Differential Equation (ODE) to find out the S, I and R. Solving ODE requires a steady state theorem to be used whereby the total number of susceptible, infected and recovered for population in Africa within a time period can be immediately identified. Thus, the progression of River blindness disease in the future can be predicted by using Euler method in Matlab software and thus leading the authority to conduct a control practice. Without the control practice, this disease may become worst and as such increasing the number of susceptible, infected and also non-recovery victims in Northern Nigeria, Africa.

2. MATERIALS AND METHOD

2.1 Methodology

This project considered using the SIR model and derivation of ordinary differential equation (ODE) to find out the S, I and R. The research methodology include the following steps derived from the model of SIR.

STEP 1 Applying the SIR model

STEP 2 Identifying the parameter values

STEP 3 Solving the general endemic SIR model using the Euler Method

STEP 4 Analyzing the endemic steady-state for S, I and R

2.2 Derived Model

The ODE solution should use the SIR model steady state theorem, in which the number of susceptible, infected and recovery cases for the population in Northern Nigeria, Africa can be identified over time. The progression of River blindness disease in the future can be predicted using the Euler method in Matlab and in turn would lead to a control practice. Without the control of the disease, River blindness may be the worst and increase the number of susceptible, infected and also non-recovery victims in Northern Nigeria, Africa.[18]

The SIR model with vital dynamic is used in endemic to calculate the number of susceptible, infected and recovery individuals in a population. This model has been selected in line with the nature and progression of the disease in the population with some assumptions. Firstly, the model is dynamic population, with individuals born into the system will be susceptible and the individuals die due to non-recovery or natural death that may occur. Then, individuals who come into the system will be susceptible and just leave the compartment with the infection. Next, the infected individuals will enter the recovery compartment and become immune. Only births and deaths are considered without consideration of the infected population affected by age, gender, social status and race. In addition, there is no immunity inherited in the population and the population is homogeneous mixed base at interaction level. Finally, there is no change in population of black fly.

2.1.1 STEP 1 Applying the SIR model

In this case, it has the inclusion of newborns into susceptible and deaths in the population, expressed as births and deaths rate, thus μ will affect the value of susceptible, infected and recovered individuals population in a time. Assume that the population size N holds a relation:

$$N = S(t) + I(t) + R(t) \quad (1)$$

Based on Kermack & McKendrick (1927), the general endemic SIR model with vital dynamic given by equation:

$$\frac{dS}{dt} = \mu N - \mu S - \beta IS \quad (2)$$

$$\frac{dI}{dt} = \beta IS - \gamma I - \mu I \quad (3)$$

$$\frac{dR}{dt} = \gamma I - \mu R \quad (4)$$

where,

S: The percentages of susceptible individuals

I: The percentages of infected individuals

R: The percentages of recovered individuals

N: Total population

β : The contact rate between the susceptible and infected individuals

γ : The recovery rate of infected individuals

μ : Births and deaths rate of population

t: The observed time

STEP 2 Identifying the parameter values

In order to calculate the contact rate between the susceptible and infected individuals, β and recovery rate of the infected individuals, γ , thus Tassier (2013) stated that the parameters are defined as follows :

$$\gamma = \frac{1}{p} \tag{5}$$

$$\beta = \frac{q}{t} \tag{6}$$

where,

p: The period for an individual to recover after infection

q: Total births and deaths in the population

t: The observed time period

STEP 3 Solving the general endemic SIR model using the Euler Method

According to Ochoche (2007), the general equation of the Euler Method is given :

$$y_{n+1} = y_n + hk_1 \tag{7}$$

The variables and parameters value will be substituted in the Euler Method that will be modified. The numerical analysis will be applied to identify the behavior of River blindness disease within 90 days and to predict the progression after 90 days.

STEP 4 Analyzing the endemic steady-state for S, I and R

From the equation (3), based on endemic steady state theory using the SIR model, set the infection rate $\frac{dI}{dt} = 0$. From $\beta SI - \gamma I - \mu I = 0$, solve it until the steady state of susceptible individuals. S will be obtained,

$$S = \frac{\gamma + \mu}{\beta} \tag{8}$$

By taking the equation (2) , set the susceptible rate, $\frac{dS}{dt} = 0$. From $\mu - \mu S - \beta SI = 0$, derive the equation by replacing the $S = \frac{\gamma + \mu}{\beta}$. Then, the steady-state of infected individuals I is obtained,

$$I = \frac{\mu}{\beta} \left(\frac{\beta}{\gamma + \mu} - 1 \right) \tag{9}$$

Next, using the function $N=S+I+R$, where in this project, the value for each S, I and R is measured in term of probability by assuming the total population N is 1. Thus, the steady state of recovered individuals R to be $R = 1 - S - I$. From equation (4) and replacing for $S = \frac{\gamma + \mu}{\beta}$

and $I = \frac{\mu}{\beta} \left(\frac{\beta}{\gamma + \mu} - 1 \right)$, then the R is obtained,

$$R = 1 - \frac{\gamma + \mu}{\beta} - \frac{\mu}{\beta} \left(\frac{\beta}{\gamma + \mu} - 1 \right) \tag{10}$$

STEP 5 Applying steady-state of S, I and R to find basic breeding analysis

Generally, R_0 is the number of basic breeding and $R_0 = \frac{\beta}{\gamma + \mu}$. From the steady state of S, I and R, the basic breeding in term of R_0 of S^* , I^* and R^* is determined,

$$S^* = \frac{1}{R_0} \tag{11}$$

$$I^* = \frac{\mu}{\beta} (R_0 - 1) \tag{12}$$

$$R^* = 1 - \frac{1}{R_0} - \frac{\mu}{\beta} (R_0 - 1) \tag{13}$$

where;

R_0 : The number of basic breeding disease

S : The number of susceptible individuals for second infection.

I : The number of infected individuals for second infection.

R : The number of recovered individuals for second infection.

3. LOCAL STABILITY ANALYSIS

The progression of River blindness disease within 90 days using the Euler Method in Matlab :

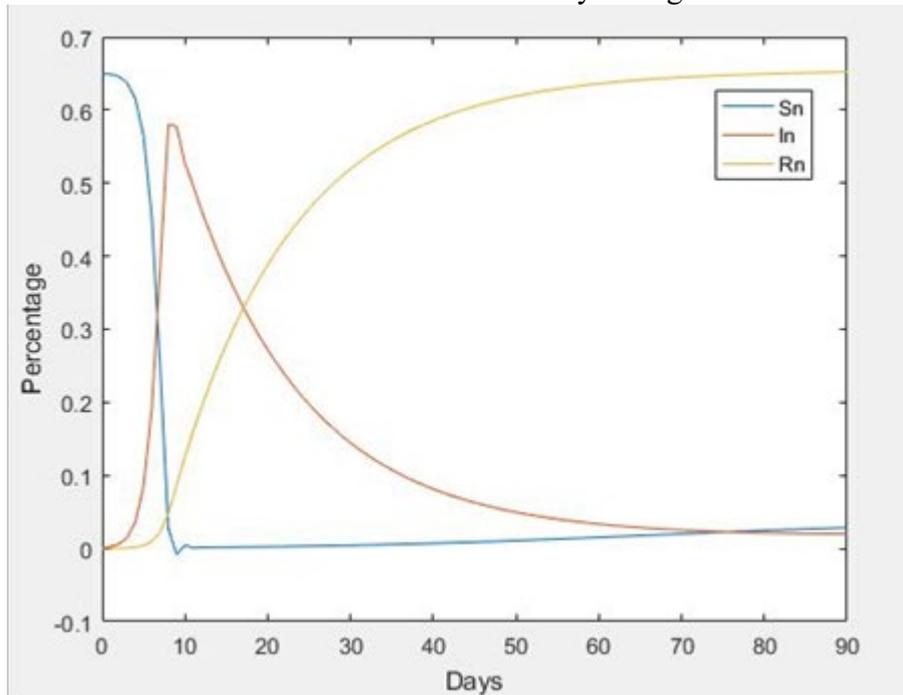


Figure 3.1 Graphical Result of River blindness disease of population within 90 days

The graph result of the input variables and parameters is shown in Figure 3.1 for the percentage change for susceptible, infected and recovered individuals of the population in Northern Nigeria, Africa. Based on Figure 3.1, within 30 days, there are significant changes to the percentage for susceptible and infection in the population. For each percentage change of susceptible, the infected and recovered rate for the population are then analysed and the reason for the change is indicated as below:

($0 < t < 20$) days

There is a point of intersection between the susceptible and infected lines estimates between 6 to 8 days before the susceptible percentage continue to decrease sharply until it reaches a minimum point. This is because most of the susceptible individuals have been infected and led the percentage of infected individuals to rise sharply within 10 days reaching the maximum point. Meanwhile, the intersection means that within a certain period of time, the number of susceptible individuals is equal to the number of infected in the population. The percentage of

the recovered individuals begins to increase as the period for an individual to recover after infection is within 15 days.

From the graph, estimated between 8 to 10 days, there is a crossing point between the susceptible and recovery lines. This is because the number of susceptible individuals is equal to the number of recovered in the population. During this period, the percentage of infected individuals will begin to decline as more numbers have been recovered, thus contributing to the increasing percentage of recovered individuals.

($20 < t < 90$) days

Based on the graph, on this period, once again there is a point of intersection at the susceptible and infected lines between 75 to 77 days. The percentage for susceptible individuals has increased slightly because of the increasing number of population, by which an individual that is born will be classified as a susceptible individual. Moreover, the infected individuals might decrease due to death caused by infection, death during the infection period or having recovered from the disease.

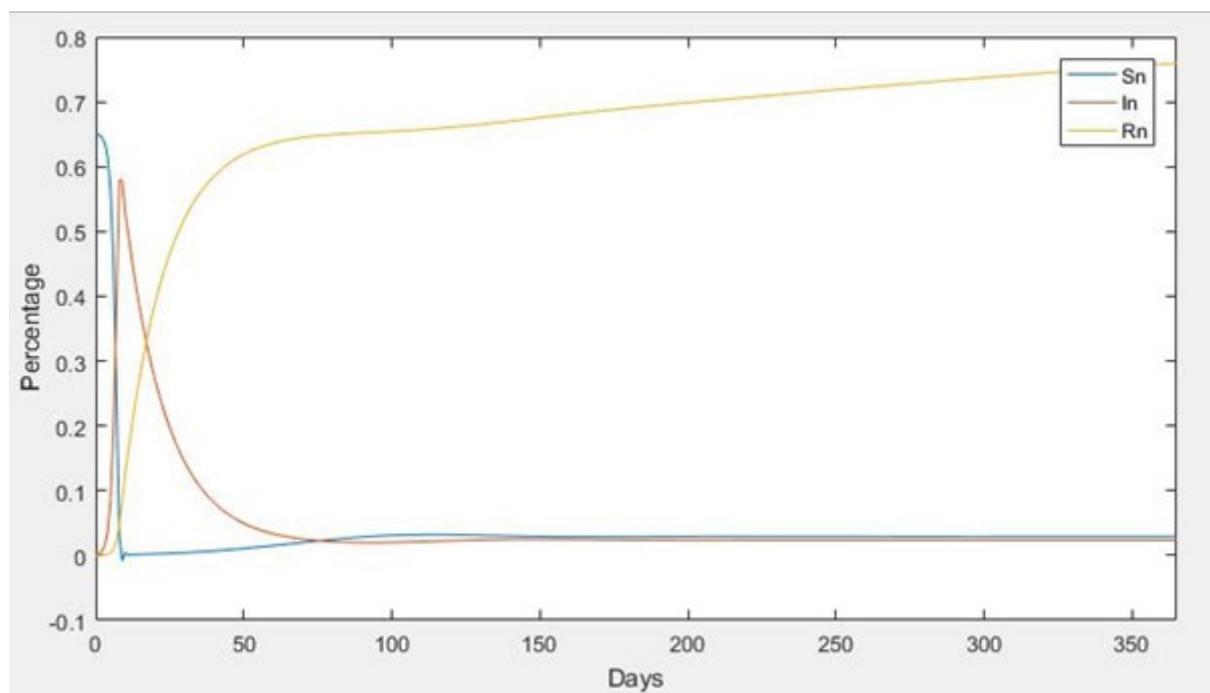


Figure 3.2 Graphical Result

The graph result of the input variables and parameters has been shown in Figure 5.1 for the percentage change for susceptible, infected and recovered individuals of the population in Northern Nigeria, Africa. Based on Figure 5.1, within 30 days, there are significant changes to the percentage for susceptible and infection in the population. For each percentage change of susceptible, the infected and recovered number for the population have been analyzed and the reason for the change indicated as below.

After 90 days

The progression of River blindness disease for 365 days or 1 year has been predicted and shown in Figure 3.2. The percentage of susceptible and infected individuals did not show a significant difference and has almost produced a straight line estimated after 90 to 365 days. Based on the graph, an observation made on the percentage for susceptible individuals showed that it is still slightly higher than the percentage of infected individuals of the population. However, the disease still does not die out since the percentage of infected individuals remains greater than 0. The percentage of recovered population is still rising as the most of infected population is fully recovered. In addition, the value of the percentage for susceptible, infected and recovered individuals in the population for the first 30 days in Figure 8 is more detailed and has been estimated utilising the Euler Method using Microsoft Excel. The numerical data in Table 1 is attached to the Appendix section. With the observation of the values of each S, I, and R more specifically, it is evident that the disease is endemic to the population of Northern Nigeria and Africa.

Based on the value of S, I, R, the disease progression can be seen in the term R_0 . Based on the research by Jacquez and O'Neill (1991), the number of basic breeding R_0 is useful to determine whether infectious diseases can spread through the population or not. Generally, if $R_0 < 1$, the infection will die in the long term, while if $R_0 > 1$ infection will spread in the population.

4. RESULT AND DISCUSSION

In conclusion, based on the SIR model, the development of the River Blindness disease among the population of Northern Nigeria, Africa is identified in more detail within 90 days. The development of the disease in the future is estimated by solving it using the Euler method in the simulation tools, Matlab. Based on the results obtained, the number of infected individuals still did not reach the zero value after 90 days. This means the disease is endemic in vital dynamic (births and deaths) and will continue to infect the population in the long term. The percentage of recovered population is still rising as the most of infected population is fully recovered after 90 days. This is caused by the improvement of prevention and treatment of the River blindness disease in the population causing the recovery number to decline. As the disease progression is identified, this facilitates disease control methods to reduce the number of susceptible, infected and recovered individuals will be successful.

Compared to other research, the results of analysis of previous work (Stolk et al., 2015, Coffeng et al., 2014a, Turner et al., 2014) suggest that whether the 2020/2025 onchocerciasis elimination goals will be met critically or not depended on when an intervention began and how intensively and effectively it was implemented (e.g. annual or biannual MDA with ivermectin; levels of coverage) and on the local transmission conditions. To conclude, in areas where the prevailing ecological conditions are highly propitious to transmission – including many foci in West Africa the initial endemic prevalence have been found to be very high.

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