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ANALYSIS OF PARTICULATE MATTER (PM10) IN MALAYSIA USING MARKOV CHAIN MODEL

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ABSTRACT

These Particulates matter (PM_{10}) received great attention due to its potential to cause adverse health effects. This study proposes the application of Markov chain model onto extreme PM_{10} concentration. Data of daily maximum PM_{10} concentration for fifteen years at ten different locations of the monitoring stations selected at random was observed. Transition probability matrix was determined and Markov chain model was fitted. Chi-square test for independence was conducted. States of transition are dependent of the observatory stations and the state transition chain does follow Markov chain property. The behavior of the transition state shows that Pasir Gudang, Petaling Jaya and Melaka recorded to have the highest value of PM_{10} concentration while Kota Kinabalu has recorded the healthiest Air Pollutant Index (API) value throughout the past fifteen years. The number of transitions of days for the value of PM_{10} concentration to be in its steady-state for each station is between 36 to 79 days.

Keywords: *markov chain model; particulate matter (PM_{10}); API value; transition probability matrix; limiting probability.*

1. INTRODUCTION

The main concern of this study is mainly based on PM_{10}, which has been classified as the most significant pollutant in Southeast Asia including peninsular Malaysia (Juneng et al, 2009). This particulate air pollution is a mixture of solid, liquid or solid and liquid particles suspended in the air which vary in size, composition and origin. Vehicle emissions, power stations and industrial sectors are the three main contributors of PM_{10} in Malaysia. As the most prevalent pollutant in Malaysia, PM_{10} concentration was frequently recorded exceeded the safe value of Recommended Malaysia Ambient Air Quality Guideline (RMAAQG), especially during the dry season while high PM_{10} concentrations were detected during dry season or also known as summer monsoon (June to September) due to the vast quantities of smoke releases by biomass burning from regional sources (Mohamed Noor et al, 2015). New Ambient Air Quality Standard was established in order to replace the older Malaysia Ambient Air Quality Guideline that has been used since 1989. The one of the concerns of the air pollution studies is to compute the concentrations of one or more pollutants' species in space and time in relation to the independent variables, for instance emissions into the atmosphere, meteorological factors and parameters. Air pollutants are released to the ambient air by two major sources namely, natural sources and anthropogenic activities (Environment, 2016).

2. MATERIAL AND METHOD

2.1 Data Description

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Data uses in this study are obtained from Department of Environment Malaysia. The data collected was from January 1, 2000 until December 31, 2014. This study covers ten locations of the monitoring stations selected at random, namely Kuching, Petaling Jaya, Kuantan, Ipoh, Kota Kinabalu, Seremban, USM, Kota Bahru, Melaka and Pasir Gudang. In this study, Markov chain model was adopted to model the daily maximum air pollutant PM_{10} concentration. Markov chain model includes classifying the data into states of transition performing test of independence, developing the count matrix into transition probability matrix and iterating of the matrix until it reaches equilibrium.

2.2 Markov Chain Model

This project considered the Markov chain models to analyze the probability of having a pollutant's concentration belonging to a given interval. A particular case of this problem is the study of the probability that a given environmental standard is surpassed.

Markov chain is a sequence of random variables X_0, X_1,... with values in a countable set S if at any time n, the future states (or values) $X_{n+1,}X_{n+2}$, ... depend on the history $X_0, \dots, X_n$ only through the present state X_n. Fundamental stochastic processes that have many diverse applications are also known as Markov chains. This is because a Markov chain represents any dynamical system whose states satisfy the recursion $X_n = f\ (X_{n-1}, Y_n), n \geq 1$, where $Y_1, Y_2, \dots$ are independent and identically distributed (i.i.d.) and f is a deterministic function. That is, the new state X_n is simply a function of the last state and an auxiliary random variable (Serfozo, 2009).

2.3 State of transition

The state of transition must be known before sorting the data. This is the most crucial part as it will decide the future result in the long term run. The API index was used as a scale to determine the states of transition. However, this study uses six states of transitions as shown in Table1.

Table 1: Index to fit as transition state

State	Range Value of API
Good (G)	0 – 50
Moderate (M)	51 – 75
Less moderate (L)	76 – 100
Unhealthy (U)	101 – 200
Very unhealthy (V)	201 – 300
Hazardous (H)	Above 300

2.4 Transition probability matrix

We shall assume that the state space $S = \{0, 1, 2, \dots\}$, the integers, or a proper subset of the integers.

A stochastic process $X = \{X_n : n \geq 0\}$ on a countable set S is called a Markov chain if for all times $n \geq 0$ and all states $i_0, \dots, i, j \in S$,

$$P\{X_{n+1} = j \mid X_0, \dots, X_n\} = P\{X_{n+1} = j \mid X_n\}, \qquad (3.5.1)$$

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$$P\{X_{n+1} = j \mid X_n = i\} = P_{ij}. \qquad (3.5.2)$$

Transition Probability matrix can be obtained from transition frequency matrix. The one-step transition matrix is the probability of transitioning from one state to another in a single step. The Markov chain is said to be time homogeneous if the transition probabilities from one state to another are independent of time, *n*. The transition probability matrix, *P*, is the matrix consisting of the one-step transition probabilities, P_{ij}. The P_{ij} is the probability that the Markov chain jumps from state *i* to state *j*. These transition probabilities satisfy $\sum_{j\epsilon S} P_{ij} = 1, i \in S$, and the matrix $P = [P_{ij}]$ is the transition matrix of the chain. Condition (3.5.1), called the Markov property, says that, at any time *n*, the next state X_{n+1} is conditionally independent of the past $X_0, \dots, X_{n-1}$ given the present state X_n. In other words, the next state is dependent on the past and present only through the present state. The Markov property is an elementary condition that is satisfied by the state of many stochastic phenomena. Condition (3.5.2) simply says the transition probabilities do not depend on the time parameter n, the Markov chain is therefore "time-homogeneous". If the transition probabilities were functions of time, the process X_n would be a non-time-homogeneous Markov chain. Such chains are like time-homogeneous chains, but the time dependency introduces added accounting details that we will not address here (Serfozo, 2009).

2.5 Classification of States of a Markov Chain

A state can be an absorbing state, once the system enters that system; it would remain there forever, or from a state transition of the system to another state is possible. If two states *i* and *j* are such that $P_{ij}^{(m)} > 0$ for some $m \geq 0$ and $P_{ij}^{(n)} > 0$ for some $n \geq 0$, the two states can be reached from one another in any number of steps, then *i* and *j* are called communicative states $i \leftrightarrow j$ (Goyal, 2012).
A state is said to be recurrent if, any time that we leave that state, we will return to that state in the future with probability one. On the other hand, if the probability of returning is less than one, the state is called transient. For any state *i*, we define
$f_{ii} = P(X_n = i, \text{for some } n \geq 1 \mid X_0 = i)$.
State *i* is recurrent if $f_{ii} = 1$, and it is transient if $f_{ii} \leq 1$.

2.6 Limiting Probability

Limiting probability is that the process will be in state *j* after a large number of transitions, and this value in independent of the initial state. In general, it seems that, as $n \to \infty$, $P_{ij}^{(n)}$ is converging to some value which is the same for all starting state *i*. indeed, it turns out that under certain conditions, the *n*-step transition probabilities approach a number, π_j, that is independent of the starting state *i*, that is, $P_{ij}^{n} \to \pi_j$. Concerning a regular Markov chain, the most important fact is the existence of a limiting probability distribution $\pi = (\pi_0, \pi_1, \dots, \pi_N)$ where $\pi_j > 0$ for *j* = 0, 1,…, *N* and $\sum_j \pi_j = 1$, and this distribution is independent of the initial state. After a long time, the initial state is forgotten and $\lim_{n\to\infty} P_{ij}^{n} = \pi_j$, where π_j is the limiting distribution that is independent of the starting state *i* (Pishro-Nik, 2016). Formally, for a regular transition probability matrix $= |P_{ij}|$, we have the converge

$$\lim_{n\to\infty} P_{ij}^{n} = \pi_j > 0, \text{ for } j = 0, 1, \dots, N,$$

Or, in terms of Markov chain $\{X_n\}$,

$$\lim_{n\to\infty} P(X_n = j \mid X_0 = i) = \pi_j \text{ for } j = 0, 1, \dots, N.$$

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For an irreducible, ergodic Markov chain, $\lim_{n\to\infty} P_{ij}^{n}$ exists and is independent of i. furthermore, letting

$$\pi_j = \lim_{n\to\infty} P_{ij}^{n}, \quad j \geq 0.$$

Then π_j is the unique non-negative solution of

$$\pi_j = \sum_{i=0}^{\infty} \pi_j \; P_{ij}^{n}, \quad j \geq 0,$$

$$\sum_{j=0}^{\infty} \pi_j = 1 .$$

3. RESULT AND DISCUSSION

3.1 Data Classification and Independence Test

The daily maximum value of PM_{10} concentration obtained from the data will be classified into 6 different ranges of states as stated. Table 2 shows the frequency data of 15 years from 2000 - 2014. The result shows that the highest PM_{10} concentration in Malaysia is between 0 - 50 $\mu g/m^3$which is in a good state. Department of environmental Malaysia has stated that the level of pollution which is in between the good state has no ill effects on health and low pollution.

Table 2: Summary of the sorted data according to states of transition

	STATES OF TRANSITION					
STATION	Good (G)	Moderate (M)	Less Moderate (L)	Unhealthy (U)	Very Unhealthy (V)	Hazardous (H)
Kuching	4282	1058	110	29	0	0
Petaling Jaya	2491	2751	189	40	2	3
Kuantan	4912	540	22	5	0	0
Ipoh	3240	2143	84	12	0	0
Kota Kinabalu	4661	801	17	0	0	0
Kota Bahru	3909	1532	36	2	0	0
Pasir Gudang	2520	2834	102	20	1	2
Seremban	3566	1778	99	36	0	0
Melaka	3622	1720	100	33	2	2
Pulau Pinang	4417	969	83	10	0	0

In order to identify whether the states of transition are dependent to the observatory station, a test of independence has been conducted. It is statistically proven that the states of transition are dependent of the observatory stations. The next state is dependent on the past and present only through the present state. Hence, the state transition chain of maximum daily PM_{10} concentration does follow the Markov chain property. Table 3 shows the transition frequency matrix of maximum daily PM_{10} concentration for all stations from 2000 – 2014.

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Table 3: Transition Frequency Matrix of Max Daily PM_{10} concentration for all stations

	Good (G)	Moderate (M)	Less Moderate (L)	Unhealthy (U)	Very Unhealthy (V)	Hazardous (H)
Good (G)	33981	3628	11	0	0	0
Moderate (M)	3621	12151	342	15	0	0
Less Moderate (L)	16	329	439	58	0	0
Unhealthy (U)	2	21	50	110	3	1
Very Unhealthy (V)	0	0	0	1	2	2
Hazardous (H)	0	0	0	3	0	4

3.2 Transition frequency matrix

An array of numbers describing the rate of a continuous time Markov chain moves between the states of transition is known as transition frequency matrix. Table 4 shows transition frequency matrix for each ten stations.

Table 4**:** Transition frequency matrix for each ten stations

KUCHING

$$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 4018 & 264 & 0 & 0 \\ 264 & 761 & 32 & 1 \\ 0 & 33 & 70 & 7 \\ 0 & 0 & 8 & 21 \end{bmatrix} \end{array}$$

PETALING JAYA

$$\begin{array}{cc} & \begin{array}{cccccc} G & M & L & U & V & H \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \\ V \\ H \end{array} & \begin{bmatrix} 2022 & 468 & 1 & 0 & 0 & 0 \\ 468 & 2200 & 82 & 4 & 0 & 0 \\ 1 & 80 & 98 & 10 & 0 & 0 \\ 1 & 5 & 9 & 24 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

KUANTAN

$$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 4734 & 178 & 0 & 0 \\ 178 & 352 & 10 & 0 \\ 0 & 9 & 11 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix} \end{array}$$

IPOH

$$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 2834 & 406 & 0 & 0 \\ 405 & 1699 & 36 & 3 \\ 1 & 35 & 45 & 3 \\ 0 & 3 & 3 & 6 \end{bmatrix} \end{array}$$

USM

$$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 4155 & 258 & 4 & 0 \\ 255 & 684 & 28 & 2 \\ 7 & 25 & 47 & 4 \\ 0 & 2 & 4 & 4 \end{bmatrix} \end{array}$$

KOTA BAHRU

$$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 3443 & 466 & 0 & 0 \\ 465 & 1054 & 13 & 0 \\ 1 & 12 & 22 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

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MELAKA	**PASIR GUDANG**
$\begin{array}{cc} & \begin{array}{cccccc} G & M & L & U & V & H \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \\ V \\ H \end{array} & \left[\begin{array}{cccccc} 3215 & 406 & 1 & 0 & 0 & 0 \\ 405 & 1270 & 42 & 3 & 0 & 0 \\ 1 & 41 & 47 & 11 & 0 & 0 \\ 1 & 3 & 10 & 18 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right] \end{array}$	$\begin{array}{cc} & \begin{array}{cccccc} G & M & L & U & V & H \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \\ V \\ H \end{array} & \left[\begin{array}{cccccc} 2027 & 492 & 0 & 0 & 0 & 0 \\ 491 & 2282 & 58 & 2 & 0 & 0 \\ 1 & 55 & 38 & 8 & 0 & 0 \\ 0 & 4 & 6 & 9 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right] \end{array}$
KOTA KINABALU	**SEREMBAN**
$\begin{array}{cc} & \begin{array}{ccc} G & M & L \end{array} \\ \begin{array}{c} G \\ M \\ L \end{array} & \left[\begin{array}{ccc} 4405 & 255 & 1 \\ 255 & 539 & 7 \\ 1 & 7 & 9 \end{array}\right] \end{array}$	$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \left[\begin{array}{cccc} 3141 & 421 & 4 & 0 \\ 422 & 1322 & 34 & 0 \\ 3 & 32 & 52 & 12 \\ 0 & 3 & 9 & 24 \end{array}\right] \end{array}$

3.3 Transition Probability Matrix

The transition probability matrices were all computed from the obtained transition frequency matrix, M. The behavior of daily maximum PM_{10} concentration at each station can be observed from the individual transition frequency matrix as shown below in Table 5.

Table 5: Transition probability matrix for each ten stations

STATION	TRANSITION PROBABILITY MATRIX, P
KUCHING	$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \left[\begin{array}{cccc} 0.9383 & 0.0617 & 0.0000 & 0.0000 \\ 0.2495 & 0.7193 & 0.0302 & 0.0009 \\ 0.0000 & 0.3000 & 0.6364 & 0.0636 \\ 0.0000 & 0.0000 & 0.2759 & 0.7241 \end{array}\right] \end{array}$
PETALING JAYA	$\begin{array}{cc} & \begin{array}{cccccc} G & M & L & U & V & H \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \\ V \\ H \end{array} & \left[\begin{array}{cccccc} 0.8117 & 0.1879 & 0.0004 & 0.0000 & 0.0000 & 0.0000 \\ 0.1699 & 0.7988 & 0.0298 & 0.0015 & 0.0000 & 0.0000 \\ 0.0053 & 0.4233 & 0.5185 & 0.0529 & 0.0000 & 0.0000 \\ 0.0250 & 0.1250 & 0.2000 & 0.6000 & 0.0250 & 0.0250 \\ 0.0000 & 0.0000 & 0.0000 & 0.5000 & 0.5000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.3333 & 0.0000 & 0.6667 \end{array}\right] \end{array}$
KUANTAN	$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \left[\begin{array}{cccc} 0.9638 & 0.0362 & 0.0000 & 0.0000 \\ 0.3296 & 0.6519 & 0.0185 & 0.0000 \\ 0.0000 & 0.4091 & 0.5000 & 0.0909 \\ 0.0000 & 0.2000 & 0.2000 & 0.6000 \end{array}\right] \end{array}$

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IPOH	$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 0.8747 & 0.1253 & 0.0000 & 0.0000 \\ 0.1890 & 0.7928 & 0.0168 & 0.0014 \\ 0.0119 & 0.4167 & 0.5357 & 0.0357 \\ 0.0000 & 0.2500 & 0.2500 & 0.5000 \end{bmatrix} \end{array}$
KOTA KINABALU	$\begin{array}{cc} & \begin{array}{ccc} G & M & L \end{array} \\ \begin{array}{c} G \\ M \\ L \end{array} & \begin{bmatrix} 0.9451 & 0.0547 & 0.0002 \\ 0.3184 & 0.6729 & 0.0087 \\ 0.0588 & 0.4118 & 0.5294 \end{bmatrix} \end{array}$
SEREMBAN	$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 0.8808 & 0.1181 & 0.0011 & 0.0000 \\ 0.2373 & 0.7435 & 0.0191 & 0.0000 \\ 0.0303 & 0.3232 & 0.5253 & 0.1212 \\ 0.0000 & 0.0833 & 0.2500 & 0.6667 \end{bmatrix} \end{array}$
USM	$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 0.9407 & 0.0584 & 0.0009 & 0.0000 \\ 0.2632 & 0.7059 & 0.0289 & 0.0021 \\ 0.0843 & 0.3012 & 0.5663 & 0.0482 \\ 0.0000 & 0.2000 & 0.4000 & 0.4000 \end{bmatrix} \end{array}$
KOTA BAHRU	$\begin{array}{cc} & \begin{array}{cccc} G & M & L & U \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \end{array} & \begin{bmatrix} 0.8808 & 0.1192 & 0.0000 & 0.0000 \\ 0.3035 & 0.6880 & 0.0085 & 0.0000 \\ 0.0278 & 0.3333 & 0.6111 & 0.0278 \\ 0.0000 & 0.0000 & 0.5000 & 0.5000 \end{bmatrix} \end{array}$
MELAKA	$\begin{array}{cc} & \begin{array}{cccccc} G & M & L & U & V & H \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \\ V \\ H \end{array} & \begin{bmatrix} 0.8876 & 0.1121 & 0.0003 & 0.0000 & 0.0000 & 0.0000 \\ 0.2355 & 0.7384 & 0.0244 & 0.0017 & 0.0000 & 0.0000 \\ 0.0100 & 0.4100 & 0.4700 & 0.1100 & 0.0000 & 0.0000 \\ 0.0303 & 0.0909 & 0.3030 & 0.5455 & 0.0303 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5000 & 0.5000 \\ 0.0000 & 0.0000 & 0.0000 & 0.5000 & 0.0000 & 0.5000 \end{bmatrix} \end{array}$
PASIR GUDANG	$\begin{array}{cc} & \begin{array}{cccccc} G & M & L & U & V & H \end{array} \\ \begin{array}{c} G \\ M \\ L \\ U \\ V \\ H \end{array} & \begin{bmatrix} 0.8047 & 0.1953 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1733 & 0.8055 & 0.0205 & 0.0000 & 0.0000 & 0.0000 \\ 0.0098 & 0.5392 & 0.3725 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.2000 & 0.3000 & 0.0500 & 0.0500 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.5000 & 0.0000 & 0.5000 \end{bmatrix} \end{array}$

3.4 Classification of States

That one effective way to determine the equivalence classes, and whether each equivalence class is transient or recurrent is to just start following the possible paths. The system will only be used to model small Markov chains, since the state space in this study is small enough

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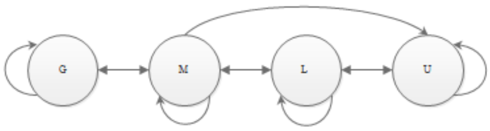
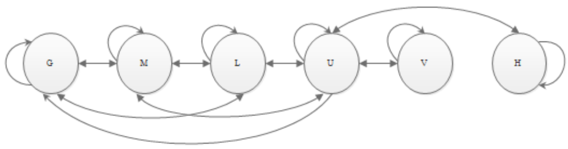
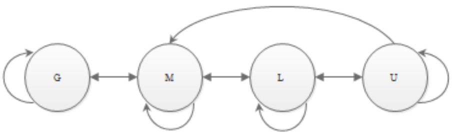
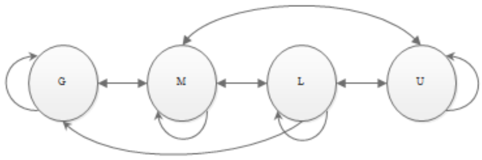
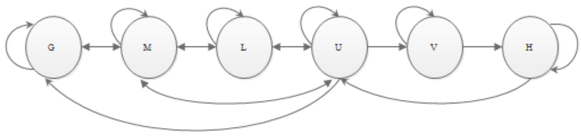
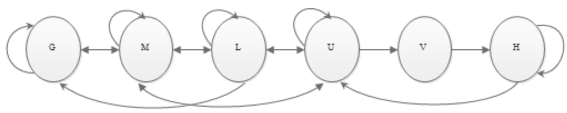
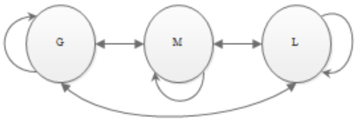
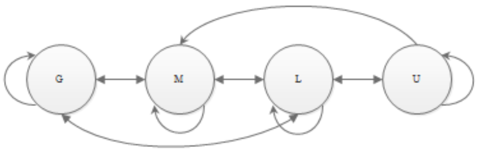
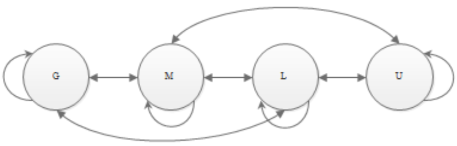
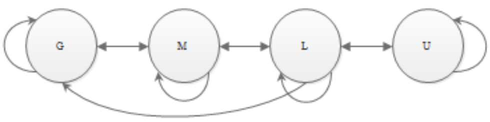
which only have six states at most, so the best way to represent them visually is as a state transition diagram. Table 6 show the state transition diagram for each station.

Table 6: State transition diagram for each ten stations

KUCHING	PETALING JAYA
	
KUANTAN	**IPOH**
	
MELAKA	**PASIR GUDANG**
	
KOTA KINABALU	**SEREMBAN**
	
USM	**KOTA BAHRU**
	

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There are all has only one communicating class where every state communicates with each other and is accessible from any other state. Therefore, their Markov chain is said to be irreducible and thus, every state is recurrent.

3.5 Limiting Probabilities

Below is a transition probability matrix, P^2 of the daily maximum PM_{10} concentration for Pasir Gudang station.

$$P^2 = \begin{bmatrix} 0.6814 & 0.3145 & 0.004 & 0.0001 & 0 & 0 \\ 0.2793 & 0.6939 & 0.0243 & 0.0025 & 0 & 0 \\ 0.1050 & 0.6528 & 0.1734 & 0.0649 & 0.0039 & 0 \\ 0.0378 & 0.4129 & 0.2509 & 0.2262 & 0.0225 & 0.05 \\ 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0.1 & 0.15 & 0.475 & 0.025 & 0.25 \end{bmatrix}$$

After 47th steps or transitions, our transition matrix will now be equal to

$$P^{47} = \begin{bmatrix} 0.4599 & 0.5173 & 0.0186 & 0.0037 & 0.0002 & 0.0004 \\ 0.4599 & 0.5173 & 0.0186 & 0.0037 & 0.0002 & 0.0004 \\ 0.4599 & 0.5173 & 0.0186 & 0.0037 & 0.0002 & 0.0004 \\ 0.4599 & 0.5173 & 0.0186 & 0.0037 & 0.0002 & 0.0004 \\ 0.4599 & 0.5173 & 0.0186 & 0.0037 & 0.0002 & 0.0004 \\ 0.4599 & 0.5173 & 0.0186 & 0.0037 & 0.0002 & 0.0004 \end{bmatrix}$$

This matrix tells us that the Markov chain is now on its steady-state or it achieved the limit probabilities and are equilibrium. This means that whether the value of PM_{10} concentration will increase or decrease from yesterday to today, the probability that it will be in good states, moderate, less moderate, unhealthy, very unhealthy and hazardous after 47 days and so on are 45.99%, 51.73%, 1.86%, 0.37%, 0.02% and 0.04% respectively. Moreover, the values of the limit probability of the Markov chain for Pasir Gudang are:
[45.99% 51.73% 1.86% 0.37% 0.02% 0.04%]
The final limit probability matrix for each station after a number of iteration until the matrix reached its equilibrium is shown below in Table 7.

Table 7: The limit probability of long-time proportion of Markov chain for each station

STATION	**STATES OF TRANSITION**					
	G	**M**	**L**	**U**	**V**	**H**
Kuching	0.7815	0.1931	0.0201	0.0053	-	-
Petaling Jaya	0.4552	0.5022	0.0345	0.0073	0.0004	0.0005
Kuantan	0.8965	0.0986	0.0040	0.0009	-	-
Ipoh	0.5913	0.3911	0.0153	0.0022	-	-
Kota Kinabalu	0.8507	0.1462	0.0031	-	-	-
Seremban	0.6508	0.3245	0.0181	0.0066	-	-
USM	0.8062	0.1769	0.0151	0.0018	-	-

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Kota Bahru	0.7135	0.2796	0.0066	0.0004	-	-
Melaka	0.6611	0.3139	0.0183	0.0060	0.0004	0.0004
Pasir Gudang	0.4599	0.5173	0.0186	0.0037	0.0002	0.0004

Figure 1 shows the number of transition of days of the maximum daily of PM_{10} concentration to be in its steady-state. Kuching takes the longest number of days for the PM_{10} concentration value to reach its steady-state which is 79 days compared to other nine stations. Meanwhile, Kota Kinabalu takes the shortest number of days which is 36 days for the PM_{10} concentration value to reach its steady-state.

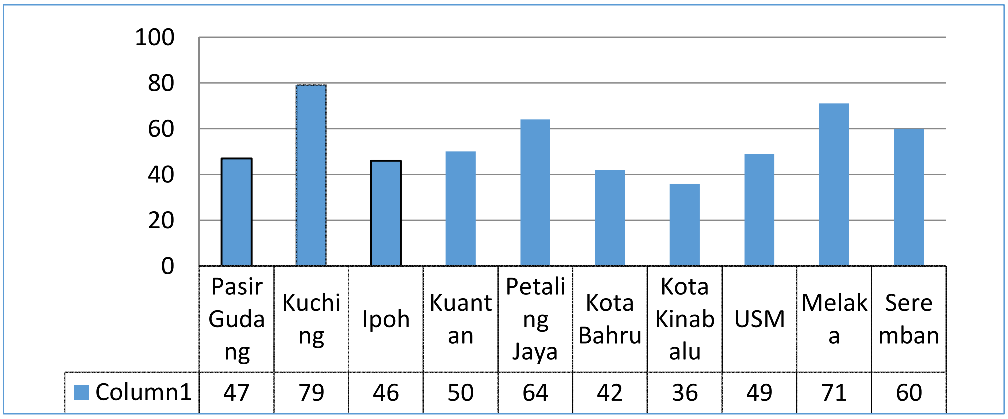

Figure 1: Number of days for each station to reach its steady state

4. CONCLUSION

Generally, PM_{10} concentration in the southwest area of peninsular Malaysia which are Pasir Gudang and Melaka, exhibits higher concentration than the east coast which are Kota Bahru and Kuantan. Moreover, Petaling Jaya that is categorized as industrial area in the southwest of peninsular Malaysia demonstrates high PM_{10} concentration that exceeded the RMAAQG value for PM_{10} that is $50\mu g/m^3$ most of the time. Rapid development and industrialization were detected as the main cause of this occurrence. In this paper we present a Markov based approach aimed at estimating the number of days for the PM_{10} concentration value to reach its steady-state or its limit probabilities. From results and discussion, Kuching takes the longest number of days for the PM_{10} concentration value to reach its steady-state which is 79 days compared to other nine stations. Meanwhile, Kota Kinabalu takes the shortest number of days which is 36 days for the PM_{10} concentration value to reach its steady-state.

Based on the results and findings of this paper, we recommend using other type of states such as interval in the significant trends of the daily maximum value of PM_{10} concentration based on the historical data. Moreover, future studies can also use different model other than Markov chain in finding the probability of the daily maximum value of PM_{10} concentration and the number of days to reach steady-states.

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