

A Manifestation of Non-Hydrodynamic Factors in Steady-State Conditions of Friction

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ABSTRACT

This study considers the mechanisms of forming the thickness of the lubricant layer with mineral and synthetic oils under a gradual increase in the rolling speed. The determined regularities reveal non-Newtonian properties acquired by the oils at high contact loads and low rolling rates. An increase in the speed and temperature is found to restore Newtonian properties of the oils. The kinetics of forming the lubricating layer can depend on the effective viscosity of oils in contact, although this parameter does not always correlate with viscosity of oils in atmospheric conditions. A delayed impact of pressure on an effective viscosity is determined at a stronger manifestation of viscoelastic properties of oils. The study reveals the existence of threshold changes in the effective viscosity at high contact loads due to the model of a free volume between the components of the lubricant.

Keywords : *lubricant layer, contact loads, friction, rheological properties, shear stress .*

Introduction

The mode of contact lubrication determines the performance and durability of rolling bearings, sliding gears, friction gears, gear couplings, and seals. Friction conditions in gears differ significantly from friction conditions in other mechanisms. Lubricants must ensure reliable operation of the equipment under conditions of prolonged oil performance in a wide temperature range under limited, elastohydrodynamic, hydrodynamic and combined modes of lubrication. Within applying the hydrodynamic theory of lubrication, the only property of oils that determines their lubricating effect is viscosity.

Viscosity of a lubricating fluid is an effective individual measure of determining the operational purpose of the oil: viscosity determines the friction loss, heat, carrying capacity, film thickness, and consumption of oil [1, 2]. At a high pressure (> 108 Pa), oil viscosity can sharply increase and acquire non-Newtonian properties [3, 4], which is not always taken into account in theoretical estimates of elastohydrodynamic lubrication (EHDL). In theoretical calculations, the pressure dependence of viscosity is set by the Barus law [5].

The impact of pressure and temperature on the effective viscosity in contact may be estimated by Roelands' equations [6].

In terms of contact, due to complex physical processes, lubricating properties of oils are very different from original characteristics – the viscosity increases hundreds of times [7, 8]. In the lubricant layer, there occur complex molecular and supramolecular changes, which are partly irreversible, leading to the appearance of a structure and a transition of a residual condition type, when oils acquire viscoelastic properties [9, 10].

In rolling bearings and gears during normal operation, the lubricant in the area of a tribological contact solidifies. This is especially true for oils with high viscosity that solidify within less than 5 micron at pressures below 400 MPa [11]. In [12], the solidifying pressures for traction oils were about 0.5 GPa, whereas those for mineral and synthetic oils ranged from 1.2 GPa to 2 GPa. Therefore, with oil viscosity dependence on pressure, a lubricant should be viewed as a nonlinear Maxwell fluid with a high-shear modulus [13, 14]. Sufficient attention should be given to changing the rheological properties of lubricants at high shear gradients. According to [15], the boundary layers of physical nature are characterized by identity of the rheological properties and the bulk liquid phase of the lubricant. However, on the contact surfaces, the formation of hemosorption films, which are arranged solid-crystal structures, increases the effective viscosity.

The selection of viscosity of a lubricant for friction units of different operational designation is based on predicting the thickness of the lubricant layer between the conjugated surfaces. It is because thickness of a lubricant

layer is the most informative indicator of the efficiency of the tribotechnical characteristics of the contact. Therefore, research on the formation of a lubricating layer as the main elastohydrodynamic factor is of interest in terms of improving the reliability of modern machines and mechanisms.

The aim of the study

The aim of the work is to study the formation of the lubricating layer and changing of the effective viscosity of mineral and synthetic oils under rolling and slip conditions for friction pairs with linear contact.

The method of determining the tribotechnical properties of the rolling friction node in sliding bearings

The research on the lubricating ability of oils was carried out on the single-contact laboratory unit SMC-2 in the mode of steady-state lubrication. The study was conducted according to a dual scheme of a roller to a roller ($d = 50 \text{ mm}$) contact in terms of sliding bearings (Figure 1).

The kinematic chain of the drive is a closed power circuit that consists of two branches connected in the contact spot of the rollers. The power circuit is driven by the direct-current aircraft generator GSK-1500 Zh, with the power being controlled by a digital converting unit.

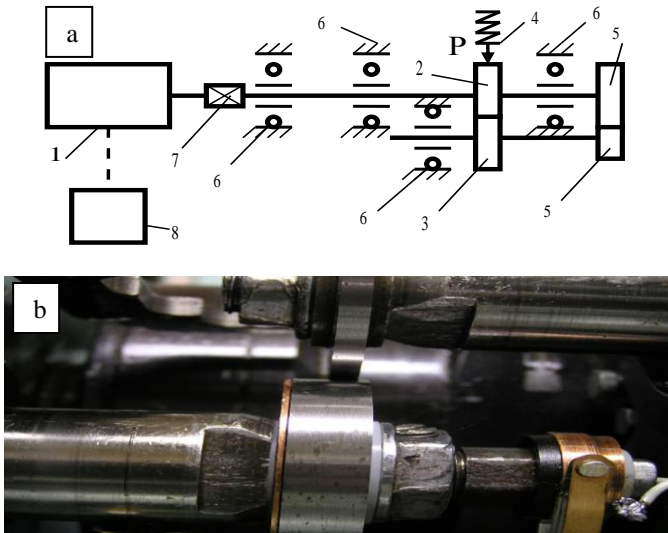


Figure 1: (a) The kinematics SMC-2: 1 - generator; 2, 3 – prototypes ; 4 - pull mechanism ; 5 - variable gear ; 6 - rolling blocks ; 7 –coupling element ; 8 –generator power digital control unit. (b) The general view of the roller-to-roller friction node.

To determine the thickness of the lubricating layer, the study involved the use of an electrical method based on measuring the drop in voltage (proportional to the layer thickness) in the mode of a normal glow discharge at a constant current strength between the contacting surfaces. In comparison with other methods, this method provides higher measurement accuracy and stability with sufficient sensitivity and continuity of registering operation parameters. This method of studying the lubricating layer was tested by, among others, M. V. Rayko, A. Cameron, P. H. Zack, and V. P. Kadomskiy [16, 17].

All measurements of the lubricating layer thickness were conducted at currents of 2 A and 4 A. In [18], the falling areas of the current-voltage characteristics were determined for the lubricating layer thickness over 4 μm ; to prevent this from happening, limiting resistance was set in the external circuit (Figure 2).

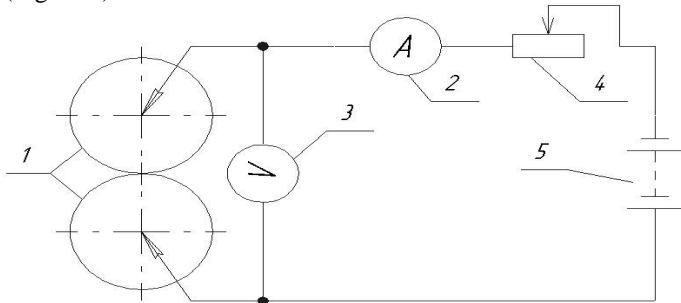


Figure 2: The electrical circuit for measuring the lubricant layer thickness in contact friction by determining the voltage drop in the mode of a normal glow discharge: 1 – The studied samples; 2 – The ammeter; 3 – The voltmeter; 4 – The regulating rheostat; 5 – The battery.

A linear dependence of voltage on the current in the electrical circuit elements is achieved at the normal constant glow discharge when only the current density changes by changing the amperage, but the voltage drop in the oil layer remains constant.

To measure the lubricating layer thickness in contact by measuring the voltage drop in the mode of a normal glow discharge, the rollers were connected to a DC circuit, and the lower roller was isolated from the circuit closure due to the weight of the machine through a textolite bearing and a fabroil ring. The electric current was supplied by a current collector mounted on the shaft of each roller. Loop current collectors of red copper wire were used to maintain a constant resistance to thousandths of ohms and to ensure operation for tens of hours without significant wear. The drum collector of the lower roller was connected to the roller with a conductor that ran through

specially cut shaft grooves. The current collector of the upper roller was the surface of this roller's shaft. To increase the accuracy of the measuring, the copper loops of the current collectors, which are prone to oxidation, should be periodically cleaned.

The electrical circuit for measuring the thickness of the lubricating layer is powered by the aircraft battery CAI-55 (with a voltage of 12 V and a capacity of 55 Ah).

Determining the thickness of the lubricating film by measuring the voltage drop in the mode of a normal glow discharge includes the following steps:

- measuring the voltage drop in the external (relative to the lubricating layer) circuits such as the parts, conductors, and current collectors (ΔU_{ext});
- determining the total voltage drop at the normal glow discharge set in the lubricant layer in contact (ΔU_{tot});
- calculating the voltage drop across the lubricating layer (ΔU_{layer}):

$$\Delta U_{\text{layer}} = \Delta U_{\text{tot}} - \Delta U_{\text{ext}} \quad (1)$$

- recalculating the values of the voltage drop across the lubricating layer into the thickness of the oil film by the set calibration tables.

To explain the established patterns of forming the lubricant layer, the rheological characteristics of oils were investigated in contact according to the techniques applied in [19].

The friction factor is calculated by the formula:

$$f = \frac{2M}{dN}, \quad (2)$$

where M is the torque, d is the diameter of the test sample, and N is the load.

The determination of the rheological characteristics of the lubricating layer includes calculation of the gradient of the shear velocity (γ) and the shear stress (τ) of the lubricating layers as well as the effective viscosity in contact (η_{eff}) by the following equations:

$$\gamma = \frac{V_{sl}}{h}, \quad (3)$$

$$\tau = \frac{f \cdot N}{S}, \quad (4)$$

$$\eta_{eff} = \frac{\tau}{\gamma}, \quad (5)$$

where V_{sl} is the sliding speed and S is the area of contact.

The study explored the lubricating and rheological properties of the additive-free mineral motor oil MC-20 (SAE-50), the distillate oil of selectively treated sulfur oil I-40 (ISO VG 68), and the synthetic polyalphaolefin oil PAO-8 (ISO VG 68). The effective oil viscosities under normal conditions (η_{20}) are 0.34 Pa·s, 0.1 Pa·s, and 0.08 Pa·s for the oils MC-20, I-40, and PAO-8, respectively.

The samples used were cylindrical rollers made of bearing steel of the Standards SAE 52100, DIN 100 Cr6, and 1.3505 (HRC = 60).

The initial surface roughness was $R_a = 0.23 \mu\text{m}$. The bulk oil temperatures were 16^0 C and 70^0 C .

The research was conducted at the contact voltage values (σ_{max}) of 250 MPa, 400 MPa, and 570 MPa under a gradual increase in the total rolling rate (from 0.48 m/s to 3.37 m/s) with a relative slip of 15 %.

The research results and discussion

Formation of the lubricant layer thickness at a gradual increase in the total rolling speed

A gradual increase in the total rolling rate from 0.48 m/s to 3.37 m/s has helped determine an increase in the thickness of the lubricant layer for the mineral oils MC-20 and I-40 and the synthetic oil PAO-8, regardless of the contact pressure and oil temperature. The formation of the base layer of an oil is $V_{\Sigma K} 1.5 \text{ m/s}$; further on, the thickness stabilizes and remains constant (Figure 3, 4).

An increase in the thickness of the lubricant layer with increasing the $V_{\Sigma K}$ to 3.37 m/s at the bulk oil temperature of 16^0 C for MC-20, I-40, and PAO-8 was, respectively, 16.5 % : 15.5 % : 15.5 % at $\sigma_{max} = 250 \text{ MPa}$ and 17 % : 13 % : 7.3 % for $\sigma_{max} = 570 \text{ MPa}$.

At the temperature of 70^0 C , there was a somewhat different pattern of growth in the lubricating film thickness in contact with an increase in the workload. When at $\sigma_{max} = 250 \text{ MPa}$ the increase in the film thickness for MC-20, I-40, and PAO-8 was similar to the percentage values registered at 16^0 C , the $\sigma_{max} = 570 \text{ MPa}$ produced an increase in the lubricant film thickness by 22.5 % : 21 % : 21 %.

With increasing the contact voltage from 250 MPa to 570 MPa for all the sample oils at the temperatures of 16⁰ C i 70⁰ C, the constant thickness of the lubricant layer is characterized by the following features (Figure 5).

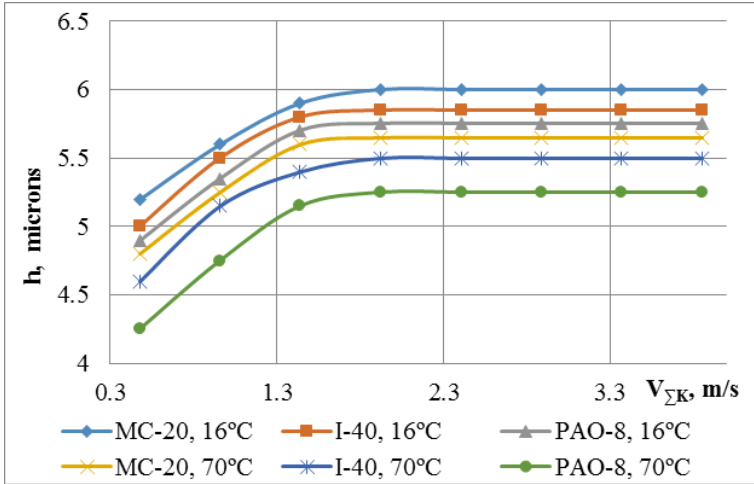


Figure 3: The formation of the lubricant layer thickness (h) in the function of the total rolling velocity ($V_{\Sigma K}$) at 250 MPa.

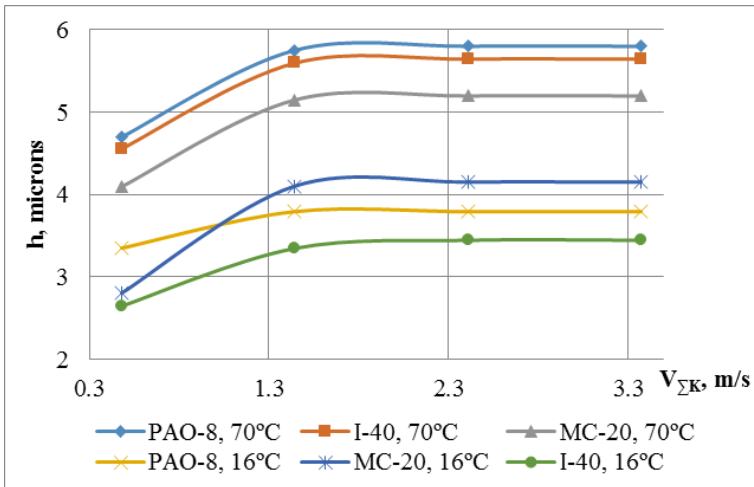


Figure 4: The formation of the lubricant layer thickness (h) in the function of the total rolling velocity ($V_{\Sigma K}$) at 570 MPa

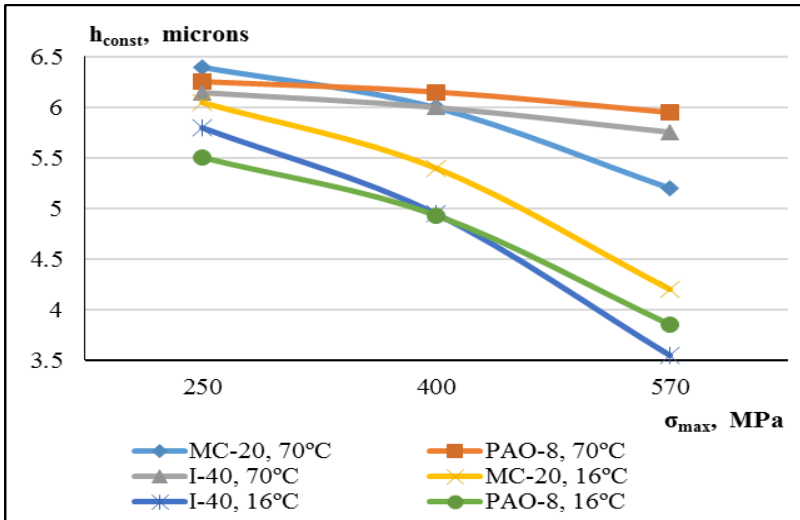


Figure 5: The dependence of the constant lubricating film thickness (h_{const}) on the contact load (σ_{max}).

At 16⁰ C, there is a clear correlation between the contact formation of the lubricant layer thickness and viscosity: the mineral oil MC–20, which is characterized by the highest viscosity under atmospheric conditions, regardless of the load, creates a lubricant film that is thicker than of the less viscous oils I–40 and PAO–8. For the synthetic oil PAO–8, in comparison with the mineral oil I–40 of similar viscosity, there is a reversion in the formation of the film thickness in contact under an increase in the workload. Thus, at $\sigma_{max} = 250$ MPa, the lubricant layer formed of PAO–8 was 5.5 % less thick than of I–40; at $\sigma_{max} = 400$ MPa, the recorded thicknesses of the films were the same; at $\sigma_{max} = 570$ MPa, the constant thickness of the lubricant layer for PAO–8 by 5.5 % exceeded the thickness for I–40.

An increased load reduces the bearing capacity of the oils, which appears to decrease the constant thickness of the lubricating layer (h_{const}) in contact; moreover, this dependence is mainly noticeable for the oils at 16⁰ C. For the oils MC–20, I–40, and PAO–8, an increase in the contact voltage from 250 MPa to 570 MPa reduces the h_{const} by 33 % : 40 % : 33 %.

At 70⁰ C, the tests determined a qualitatively different pattern of forming the lubricating films of the sample oils at higher loads. With an increase from 250 MPa to 570 MPa for the oils MC–20, I–40, and PAO–8, the h_{const} respectively declined by 17.5 % : 5 % : 5 %. At $\sigma_{max} = 250$ MPa for the more viscous oil MC–20, the lubricant layer was found thicker ($h_{const} = 6.3 \mu\text{m}$), compared to I–40 ($h_{const} = 6.10 \mu\text{m}$) and PAO–8 ($h_{const} = 6.2 \mu\text{m}$);

at $\sigma_{\max} = 570$ MPa, the lubricant film in the contact spot was the thinnest for the mineral oil MC–20 ($h_{\text{const}} = 5.2 \mu\text{m}$), unlike for I–40 ($h_{\text{const}} = 5.79 \mu\text{m}$) and PAO–8 ($h_{\text{const}} = 5.9 \mu\text{m}$).

At 70°C , the synthetic oil PAO–8, compared with the mineral oil I–40 of similar viscosity and regardless of contact voltage, provides a better lubricating ability: the lubricant layer is on average thicker by 0.1 micron.

All of the tested oils under a gradual increase in the total rolling rate from 0.48 m/s to 3.37 m/s provide a hydrodynamic lubrication regime in the contact area.

This is confirmed by the criterion of the lubrication regime (λ) [11].

$$\lambda = \frac{h}{\sqrt{R_{a1}^2 + R_{a2}^2}}, \quad (6)$$

h – Lubrication layer thickness;

R_{a1}, R_{a2} – Arithmetic mean deviation of the profile.

This parameter is, respectively, 12 : 11 : 8 at the contact pressure values of 250 MPa, 400 MPa, and 570 MPa at 16°C .

At 70°C , regardless of the load, this factor is, on average, $\lambda \approx 16$ for all of the tested oils.

The experimentally determined dependence of the lubricant layer formation by the studied oils is totally consistent with the theory of EHDL at a gradual increase in the total rate of rolling: the lubricant layer can become thicker with increasing the total speed of rolling only to 10–15 m/s [20], and an increase of the load reduces the oil film thickness in contact [21]. According to the theory of EHDL, a higher viscosity of oil produces a thicker oil film [22]. However, the experimental data confirm this pattern only when the bulk temperature of the oils is 16°C . The layer of the lubricant is registered to be thicker in the frictional contact at increasing the primary bulk temperature of the oils up to 70°C , whereas a greater carrying capacity at 70°C , which is typical of the less viscous oils PAO–8 and I–40 in comparison with MC–20, contradicts the theory of EHDL.

Non-Newtonian properties acquired by the oils in contact

Contact viscosity at an increase in the load does not change as follows from the Barus equation. At $V_{\Sigma K} = 0.48$ m/s, $\sigma_{\max} = 250$ MPa, and the bulk oil temperature of 16°C , the experimental value of η_{eff} for MC–20 is found to be a sequence higher than the estimated value; for I–40, it is twice higher; for PAO–8, it coincides with the estimated value. When the oil temperature is increased to 70°C , the experimental and estimated values of η_{eff} coincide for MC–20; for I–40, the contact η_{eff} is twice reduced, and for PAO–8 it is three

times lower compared with the estimated data. At $V_{\Sigma K}$ 0.48 m/s and $\sigma_{max} = 570$ MPa, regardless of the oil temperature, the estimated values exceed the experimental average by 3–5 orders.

The changing effective viscosity of the tested oils under a contact load at $V_{\Sigma K}$ 0.48 m/s is shown in (Figure 6).

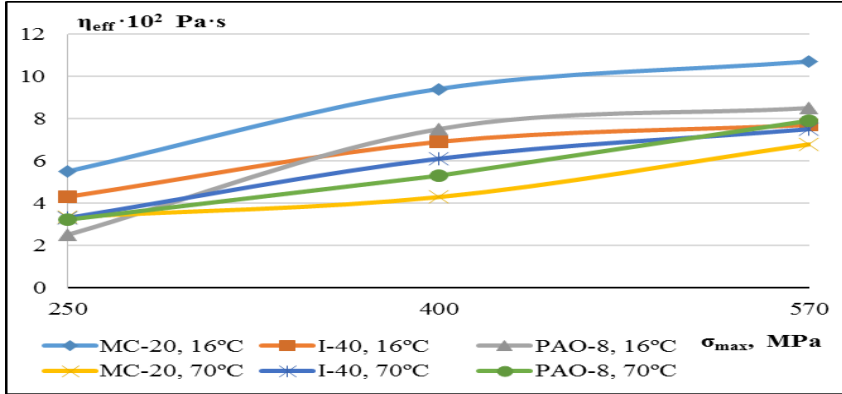


Figure 6: The dependence of the effective viscosity (η_{eff}) of the oils on the contact load (σ_{max}).

When the σ_{max} increases from 250 MPa to 570 MPa, the effective viscosity for the oils MC–20, I–40, and PAO–8 increases, respectively, 1.98 : 1.83 : 3.16 times at 16° C and 2.15 : 2.4 : 2.58 times at 70° C. For MC–20, which is characterized by a higher viscosity at atmospheric pressure, an increased load maximizes the effective viscosity compared to I–40 and PAO–8 at 16° C, regardless of the pressure.

An increase in the bulk temperature leads to a dilution of the oil, resulting in a reduced η_{eff} in contact. However, for the synthetic oil PAO–8 at $\sigma_{max} = 250$ MPa, an increasing temperature is found to increase the η_{eff} : $2.68 \cdot 10^2$ Pa·s at 16° C and $3.05 \cdot 10^2$ Pa·s at 70° C. Qualitative changes are also observed in the nature of the η_{eff} depending on the σ_{max} of the studied oils at 70° C: at $\sigma_{max} = 250$ MPa, the effective viscosity for MC–20, I–40, and PAO–8 is respectively $(3.12 : 3.11 : 3.05) \cdot 10^2$ Pa·s; with increasing the load, the η_{eff} for the oils I–40 and PAO–8 exceeds the effective viscosity of MC–20, respectively, 1.52 : 1.3 times at $\sigma_{max} = 400$ MPa and 1.1 : 1.17 times at $\sigma_{max} = 570$ MPa.

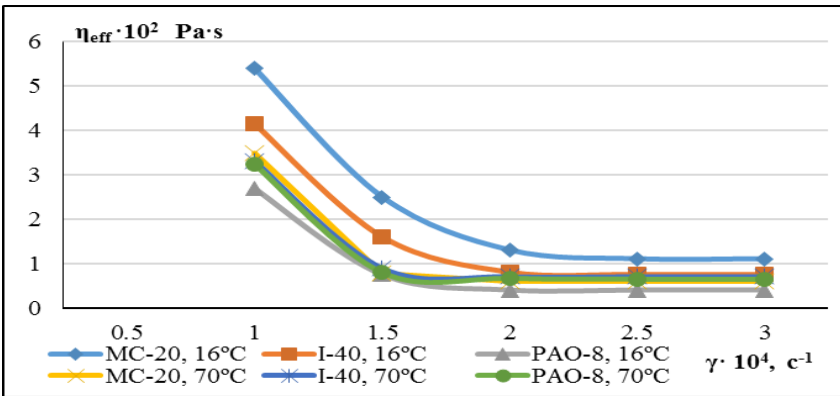
With increasing the pressure, there is less change in the viscosity of the oils, especially at 16° C. In [14], effective viscosity under an increasing load, especially in low speed rolling, is found to reach the limit value, and

then the increase of the η_{eff} slows down. This is also confirmed by the experimental results.

To explain the obtained experimental dependence of the changing viscosity effectiveness, it is essential to take into account the free volume model, which describes a number of patterns observed in liquids at an increased pressure for various bulk temperatures of oils [23]: as the pressure increases, the fluid volume is reduced by the “free volume” between the molecules while the molecules themselves retain a constant overall volume. This means that with increasing the load at a given temperature there is a limit volume of the fluid in which no molecular movements are possible. An increase in the bulk temperature to 70⁰ C causes a decline in the intermolecular energy of the molecules interaction, an increase in the potential energy, and a transition to a freer rotation [24]. These processes lead to a decrease in the η_{eff} when temperature increases, as is recorded in the presented experiments.

Thus, the study has determined a reduction in the effective viscosity with increasing the load for the oil MC–20 at 70⁰ C, compared to I–40 and PAO–8, due to a delayed impact of pressure on viscosity at a stronger manifestation of the viscoelastic properties of the tested oil [14]. Unlike I–40, the oil MC–20 in its fractional structure contains a paraffin fraction with a higher molecular weight of hydrocarbons; the content of aromatic compounds increases by 10 %, and the content of resinous components increases by 4 %.

It has been proved that, regardless of the contact temperature, the tested oils acquire the properties of non–Newtonian fluids under the effect of increased pressure; it is evident from the η_{eff} dependence on the gradient of the oil layer shear speed (Figure 7).



a

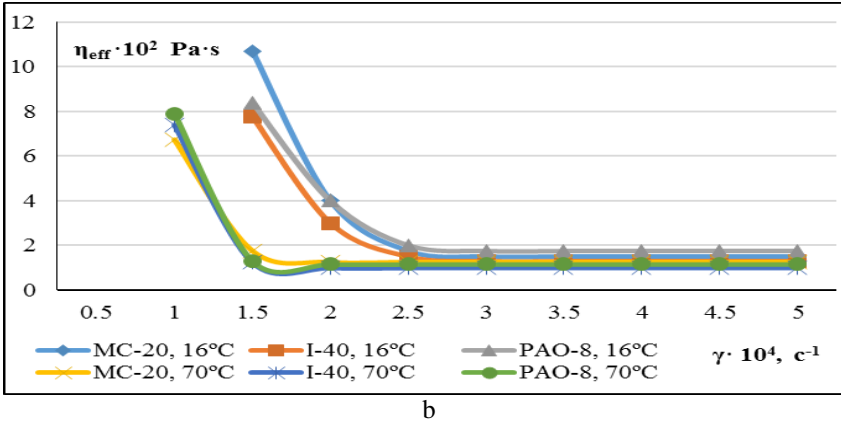


Figure 7: The dependence of the effective viscosity (η_{eff}) on the gradient of the oil layer shear speed (γ) at 250 MPa (a) and 570 MPa (b).

When the γ increases to $1.5\text{--}3.10^4 \text{ s}^{-1}$, which corresponds to a total increase in the rolling speed up to 1.5 m/s, all the tested oils reveal a reduction in the effective viscosity in contact by an average of 75-85 %; further on, the viscosity of the oils stabilizes and remains unchanged, which is typical of Newtonian fluids. The obtained data are consistent with the results of [14] on the effect of high shear gradients (about 3.10^4 s^{-1}) that lead to a de-association of the molecules and reduce the η_{eff} when there is an increase in the rolling speed for non-Newtonian fluids.

In the analysis of the η_{eff} dependence on the load at $V_{\Sigma K} 3.37 \text{ m/s}$, when the oils restore their Newtonian properties in contact, there appears a certain peculiarity (Figure 8). At 16^0 C , as σ_{max} increases from 250 MPa to 570 MPa, the increase of the η_{eff} for the oils MC-20, I-40 and PAO-8 is similar to the change in this parameter at $V_{\Sigma K} 0.48 \text{ m/s}$. However, at 70^0 C , the η_{eff} increases, respectively, 1.84 : 1.28 : 1.75 times, which on average is 1.5 times less than at $V_{\Sigma K} 0.48 \text{ m/s}$. At $\sigma_{\text{max}} = 570 \text{ MPa}$, regardless of the temperature, the mineral oils MC-20 and I-40 display a clear correlation between the η_{eff} in contact and viscosity at atmospheric pressure. Thus, the more viscous oil MC-20 is also characterized by a higher η_{eff} , compared with the less viscous oil I-40.

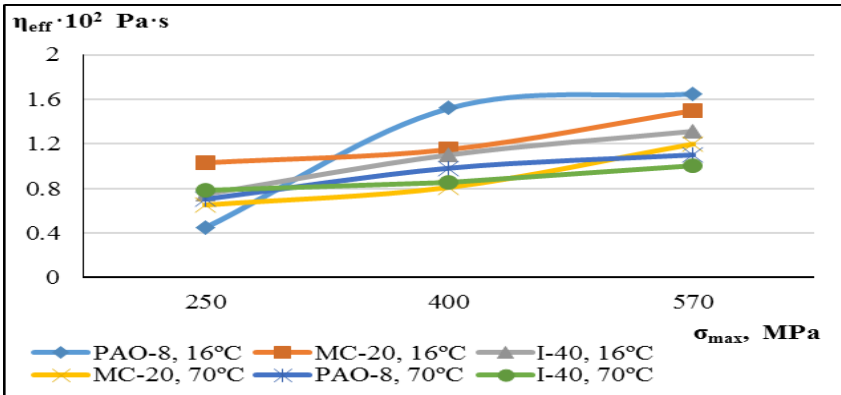


Figure 8: The dependence of the effective viscosity (η_{eff}) in the function of a contact load (σ_{max}) at $V_{\Sigma K}$ 3.37 m/s.

The undertaken analysis of the dependence $\eta_{\text{eff}} = f(\sigma_{\text{max}}, T, \gamma, \text{ and } V_{\Sigma K})$ has revealed that at $V_{\Sigma K}$ 0.48 m/s, regardless of the load and temperature, the tested oils are characterized by non-Newtonian properties. Since we have determined that the lubricant film thickness is formed under $V_{\Sigma K}$ 1.5 m/s, the bearing capacity of the oils MC-20, I-40, and PAO-8 is determined primarily by viscoelastic properties beyond the consideration of the theory of EHD.

Although petroleum oils have small compressibility, at a high pressure (≥ 400 MPa) their volume may decrease by 15–20 % [25]. In the oil MC-20 compared to the oils I-40 and PAO-8, the association of molecules is much higher: they are in larger electric fields created by neighbouring molecules, which leads to an increase of the η_{eff} at 16⁰ C and constitutes a major factor of forming the film thickness at 16⁰ C, regardless of the load. A reversion of the lubricant layer thickness, which was revealed at increasing the pressure for PAO-8 in comparison with I-40, happens due to the higher effective viscosity of the synthetic oil in the contact zone.

An increase in the bulk oil temperature to 70⁰ C results in a reduction in the energy of the intermolecular bonds, an oil dilution, and an effect of a “free volume” of the fluid. These processes reduce the oil relaxation time while increasing the efficiency of lubrication.

Conclusion

Under a gradual increase in the rate of sliding bearings, at low speeds, mineral and synthetic oils have been found to acquire the properties of non-Newtonian fluids. The lubricant layer thickness, regardless of the fractional

composition of the oil, temperature and load, is formed at a speed of up to 1.5 m/s and is determined by non-Newtonian properties of oils. The lubrication efficiency increases as temperature increases, which happens due to a decrease in the impact of non-hydrodynamic factors. At 70⁰ C, the constant thickness of the lubricant layer exceeds this parameter observed at 16⁰ C by 8 % for all oils at 250 MPa and by 20 % : 40 % : 42 % for MC–20, I–40, and PAO–8, respectively, at 400 MPa and 570 MPa. The formed lubricant layer thickness depends on the rheological properties of the oils, in particular, on their effective viscosity in contact. The study has determined that with increasing the contact load up to 570 MPa the experimental values of effective viscosity are by 3–5 orders lower than the values estimated by the Barus equation. This proves the existence of a threshold increase in the effective viscosity under conditions of increasing the pressure up to 570 MPa, which is explainable by the model of a free volume between the lubricant molecules.

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