UNIVERSITI TEKNOLOGI MARA

ON POSITIVE SOLUTIONS FOR SINGULAR BOUNDARY VALUE PROBLEMS OF DIFFERENTIAL AND DIFFERENCE EQUATIONS

NOOR HALIMATUS SA’DIAH BINTI ISMAIL

Thesis submitted in fulfillment of the requirements for the degree of
Master of Science

Faculty of Computer and Mathematical Sciences

April 2016
CONFIRMATION BY PANEL OF EXAMINERS

I certify that a Panel of Examiners has met on 20th October 2015 to conduct the final examination of Noor Halimatus Sa’diah Bt Ismail on her Master of Science thesis entitled “On Positive Solutions for Singular Boundary Value Problems of Differential and Difference Equations” in accordance with Universiti Teknologi MARA Act 1976 (Akta 173). The Panel of Examiners recommends that the student be awarded the relevant degree. The panel of examiners was as follows:

Datin Noor Habibah Hj Arshad, PhD
Professor
Faculty of Computer and Mathematical Sciences
Universiti Teknologi MARA
(Chairman)

Daud Mohamad, PhD
Professor
Faculty of Computer and Mathematical Sciences
Universiti Teknologi MARA
(Internal Examiner)

Gafurjan Ibragimov, PhD
Associate Professor
Department of Mathematics
Universiti Putra Malaysia
(External Examiner)

SITI HALIJJA SHARIFF, PhD
Associate Professor
Dean
Institute of Graduate Studies
Universiti Teknologi MARA
Date: 11th April, 2016
AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged or referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any other degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.

Name of Student : Noor Halimatus Sa'diah Bt Ismail
Student I.D. No. : 2011717203
Programme : Master of Science (by Research)
Faculty : Faculty of Computer and Mathematical Sciences
Signature of Student : [Signature]
Date : April 2016
ABSTRACT

This thesis is concerned with the existence and multiplicity of positive solutions to singular boundary value problems (BVPs) of differential and difference equations. By using the Krasnoselskii fixed point theorem on compression and expectation in cone, sufficient conditions for the existence of positive solutions are established for a singular system of first-order differential equations and singular second-order BVPs of difference equations. Our results give an almost complete structure of the existence of positive solutions for the problems studied with an appropriately chosen parameter. By choosing appropriate cone, the singularity of the equations is essentially removed and the associated positive operator becomes well defined for certain ranges of functions even when $e_1$ is negative. By employing the Krasnoselskii fixed point theorem in cone, the existence and multiplicity of positive periodic solutions for a singular system of first-order ordinary differential equations is established. As an extension, the discrete analogue of singular differential problems of second-order BVPs with a parameter is derived. The existence of positive solutions is obtained by applying the Krasnoselskii fixed point theorem in cone. The result is then extended to a singular discrete system of second-order two-point BVPs. Also the existence of positive solutions is investigated for a singular discrete system of second-order multi-point BVPs.
TABLE OF CONTENTS

CONFIRMATION BY PANEL OF EXAMINERS ii
AUTHOR’S DECLARATION iii
ABSTRACT iv
ACKNOWLEDGEMENT v
TABLE OF CONTENTS vi
LIST OF FIGURE viii

CHAPTER ONE: INTRODUCTION 1
1.1 Research Background 1
  1.1.1 Model Problems of BVPs for ODEs 1
  1.1.2 Model Problems of Discrete BVPs 2
  1.1.3 Some Examples: Real World Phenomena 3
  1.1.4 Definitions and Notations 5
  1.1.5 Review of Theorems 7
1.2 Research Problem 8
1.3 Research Objectives 9
1.4 Thesis Outline 9

CHAPTER TWO: POSITIVE PERIODIC SOLUTIONS FOR A
SINGULAR SYSTEMS OF FIRST-ORDER ODES 11
2.1 Introduction 11
2.2 Preliminary result 13
2.3 Main result 23

CHAPTER THREE: POSITIVE SOLUTIONS FOR A SINGULAR
DISCRETE SCALAR OF SECOND-ORDER TWO-POINT BVPS 30
3.1 Introduction 30
3.2 Preliminary Result 33
3.3 Main Result 41