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An Experimental and Theoretical Analysis of Projectile and Target Geometry Effects on Ballistic Limit Velocity

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ABSTRACT

FRP (Fibre Reinforce Plastics) laminates are now widely used in different industries like aviation, marine and transportation as FRP has shown considerable strength and hardness. However, these materials are vulnerable to lateral intensive static loads and impacts but have proved a good stability and strength in industrial applications. In this paper, an analytical method for ballistic impact on FRP laminates has been proposed along with impact analysis of the panel, which has been carried out in two stages; Rupture and Wave-dominated local failure. First a simple analytical model for local deformation failure and critical shear failure has been used to predict perforation in FRP laminates. For verification of the model, several projectile impact experiments have been conducted to evaluate proposed model. It is concluded that experimental data are in good agreement with proposed analytical model results and effects of different geometrical parameters like panel dimensions and projectile diameter on ballistic velocity have been studied. In addition, analytical data have also been verified with results which were obtained from of LS-DYNA simulation.

Keywords: Fibre-Reinforced Plastic, Impact, Failure Analysis, Ballistic Limit Velocity.
Introduction

Fibre-reinforced plastic (FRP) is a one type of composite materials which made of a polymer matrix reinforced with fibres. The commonly used fibres are glass, aramid or carbon. Some other fibres like paper or wood or asbestos have been rarely used and the resin usually consist of an epoxy, vinylester or polyester thermosetting plastic. Aziz has studied the ballistic impact study for the non-filled aluminium tank. The results showed that the ballistic limit for the front tank wall and rear tank wall was 257.7 m/s and 481 m/s, respectively. In addition, this study presented the correlation of the impact velocity towards residual velocity, damage area, wall deflection, velocity drop and energy absorbed [1]. Yunus has investigated, the effect of low energy impact to the residual strength and modulus of short kenaf fibre reinforced epoxy composites. He found that the low energy impacts were affecting the residual strength and residual stiffness which indicated the short kenaf fibre reinforced epoxy composites were extremely sensitive to the impact loads. The damages were manifest by the visible observation of cracks on the specimens [2]. Gu [3] has presented an analytical model of projectile penetration in composite materials which based on adsorbed energy by fibres regarding strain effect on stress-strain relation in fibres. He has modelled a woven fibre instead of a composite material. Gellert [4] obtained ballistic limit velocity on Glass Fiber Reinforced Plastic (GFRP) laminates with different thickness, under projectile impact on conic head. Ulven [5] has experimentally studied effect of projectile head shape on perforation ballistic limit and energy adsorption VARTM carbon/epoxy composite panels.

Modelling and Analysis

There are two steps that have been considered for impact analysis on FRP laminates. Complete deformation with local fragmentation and local deformation are caused by wave. First of all, an analytical model is proposed for complete laminate deformation which subjected to flat head projectile impact. This model goes with shear failure criteria which could be used to predict laminate perforation. By combining local failure model of the governing wave and general concept of critical impact velocity theory (Von Karman) second fracture mode will be gained. Assuming a predefined value for local notch, overall bending, energy adsorption and neglecting
energy losses like friction an analytical model for impact between flat head projectile and thin FRP laminates has been developed.

**Semi static force and displacement**

In Figure 1, a FRP laminate which is statically loaded with a flat projectile is shown. In the Figure 1 equivalent spring system is represented. Relation between semi static contact force $P$ with local notch is as below [6,7]:

$$P = K_c \alpha$$  \hspace{1cm} (1)

Where:

$$K_c = \frac{2R}{\pi H_0}$$  is contacting hardness;

$R$: Projectile Radius;

And $H_0$ is obtained from following [6,7]:

$$H_0 = \frac{(\gamma_1 + \gamma_2)C_{11}}{2\pi(C_{11}C_{33} - C_{13}^2)}$$

$$Q = (C_{11}C_{33} - C_{13}^2 - 2C_{13}C_{44})/2C_{11}C_{44}$$

$$r^2_{1,2} = Q \pm \sqrt{Q^2 - C_{33}/C_{11}}$$

Where $C_{ij}$ is the "Elastic Constants" for isotropic elastic material.

![Figure 1: Laminated plate encountering a flat head projectile impact](image)
Figure 2: Impact spring model

\[ P = K_{bs} W_0 + K_m W_0^3 \]  \hspace{1cm} (2)

Where:

- \( W_0 \) is vertical deflection of mid plane
- \( K_{bs} \) is shear and bending effective stiffness
- \( K_{bs} \) is effective shell stiffness for fixed plates.

\( K_m \) and \( K_b \) for fixed plates are obtained from following relations [8,9]:

\[ K_b = \frac{16\pi E_s H^3}{3(1-\nu_1^2) S^2} \]  \hspace{1cm} (3)

\[ K_m = \frac{191\pi E_s H}{162S^2} \]  \hspace{1cm} (4)

\[ K_{bs} = \frac{K_b K_s}{K_b + K_s} \]  \hspace{1cm} (5)

In the above relations, \( S \) is the laminate plate dimension. Effective shear stiffness is obtained from following relation [8]:

\[ K_s = \frac{4\pi}{3} G_{13} H \left( \frac{E_1}{E_1 - 4\nu_{12} G_{13}} \right) \left( \frac{4}{3} + \log \frac{S}{2R} \right)^{-1} \]  \hspace{1cm} (6)
Energy Adsorption

Adsorbed energy in laminate is considered as a total deformation energy, failure energy and delamination energy. Wasted energy caused by deformation is equal to sum of contact energy ($E_{ct}$) and adsorbed energy by complete deformation ($E_{bm}$). Consequently, by replacing $W_o$ (critical vertical deflection) with $W_o$ deformation energy is obtained as below:

$$E_{ad} = E_o + E_{bm} = \int_0^L Pd\alpha + \int_0^L PdW_o = \left(\frac{K_sW_{sc} + K_sW_{sc}^2}{2K_e} + \frac{1}{2}K_sW_{sc}^3 + \frac{1}{4}K_sW_{sc}^4\right)$$

By the aid of following approximation deformation energy is obtained [6].

$$P = K_{bs}W_{0f} + K_mW_{0f}^3 = 2\pi RHK\tau_{13}$$

$\tau_{13}$ is vertical laminate shear strength and $2K$ being fixture factor [6]. In this paper we consider $2K$. By considering $\phi$ as an experimental constant, adsorbed energy from failure for $H/D > \phi$ is obtained by following equation:

$$E_{fmc} = \pi R^2(H - \phi D)e_i + \pi R(\phi D)^2K\tau_{13}$$

And for:

$$E_{fmc} = \pi RH^2K\tau_{13}$$

Where $e_i$ is tensile failure energy density. The value of $\phi$ is consider 0.21 approximately [10]. Delamination is initiated under contact loading region by forming cracks in inner plane matrix by shear stress. Delamination could propagate through material complying mode 1 (tensile) and mode 2 (compressive). Delamination energy on FRP laminates is stated as below [10]:

$$E_{del} = A_d \times G_{pc}$$
In the above expression, $A_d$ is delaminating surface on mid plane and $G_{IC}$ is fracture toughness in second mode [6].

\[
E_{def} = \frac{9}{16\pi H^2} \left( \frac{P_d}{\tau_{RSS}} \right)^2 G_{IC} 
\]  

(11)

\[
P_d^2 = \frac{8\pi^2 E_i H' G_{IC}}{9(1 - \nu_i^2)} 
\]  

(12)

Where $P_d$ is delamination critical force and $\tau_{RSS}$ is internal shear stress. Energy is semi static loading in FRP laminates perforation in flat head projectile is obtained from following equation:

\[
E_T = E_{def} + E_{frac} + E_{del} 
\]  

(13)

Impact perforation energy ($E_p$) is stated as below [6]:

\[
E_p = \varphi E_T 
\]  

(14)

Where $\varphi$ is dynamic increment factor which is obtained from following equation:

\[
\varphi = \begin{cases} 
1 + B \left( \frac{V_i}{V_c} \right) & (V_i < V_c) \\
1 + B & (V_i > V_c) 
\end{cases} 
\]  

(15)

Where $B$ is experimental constant, $V_c$ is Von Karman critical velocity and $V_i$ is projectile initiate velocity. In this paper $\varphi$ is considered to be 0.21.

\[
V_s = \sqrt{\frac{2\varphi(E_{def} + E_{frac} + E_{del})}{G}} 
\]  

(16)
AN EXPERIMENTAL AND THEORETICAL ANALYSIS OF PROJECTILE AND TARGET GEOMETRY EFFECTS...

Local Failure Caused by Wave

Resisting force against flat head projectile while hitting FRP laminates are derived from following equation [18]:

\[ F = \frac{\pi D^2}{4} \left( \sigma_e + \beta \sqrt{\rho_t \sigma_e V_i} \right) \]  

(18)

Where \( \sigma_e \) is linear elastic limit for FRP laminates in compression through thickness direction, \( \rho_t \) is FRP laminate density and \( \beta \) is experimental constant which is considered for flat head projectile.

By using energy equilibrium, \( \frac{1}{2} G V_b^2 = F \cdot H \) and replacing \( V_i = V_b \) in which ballistic limit for thick FRP laminates are obtained as below:

\[ V^* = \frac{\pi \sqrt{\rho_t \sigma_e D^3 H}}{2G} \left[ 1 + \sqrt{1 + \frac{2G}{\rho_t D^3 H}} \right] \]  

(19)

Thickness Evaluation Criterion for FRP Laminates

By using \( V_b = V^* \) and replace it in equation (18) and by using \( G = \frac{1}{4} \pi D^2 L \rho_p \) in which \( L \) is length and \( \rho_p \) is projectile density, ratio of FRP laminate critical thickness to projectile diameter is obtained as below:

\[ \left( \frac{H}{D} \right)^c = \frac{\varepsilon_i^2}{2 \left[ 1 + 2 \left( \frac{E_i}{\sigma_e} \right) \left( \frac{\rho_t}{\rho_p} \right) \left( \frac{L}{D} \right) \right]} \]  

(20)

In equation (20) if \( H/D \geq (H/D)^c \) then FRP laminate will break in local mode caused by wave, otherwise it will be broken in deformation mode by local dividing in two parts.

Results and Discussion

FRP laminates are subjected to flat head projectile as shown in Figure 1. FRP laminates properties are shown in Table 1. Results obtained from proposed model are compared with experimental data which mentioned in references [4,5]. Figure 3 shows values of calculated ballistic limits in term of \( H/D \).
In this figure a comparison have been made between gathered data from experiment analysis and those data which presented in reference [4]. Critical ratio \((H/D)_c\) for GFRP laminates which are subjected to impact from a 3.84 gr and 6.35 mm flat head projectile with the aid of eq. (19) is equal to 0.160.

In Figure 4 a comparison is represented between experimental data in reference [4] and ballistic limit velocity. Critical rate ratio \((H/D)_c\) for CFRP laminates which subjected to flat head projectile which its mass is 14 gr and diameter 12.7 mm by using equation (24) is equal to 0.199.

**Table 1: Material properties**

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Carbon Epoxy</th>
<th>E-Galss/Vinylester [4,5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S)</td>
<td>100 mm</td>
<td>101.6 mm</td>
</tr>
<tr>
<td>(\rho_s)</td>
<td>1850 Kg/m^3</td>
<td>1550 Kg/m^3</td>
</tr>
<tr>
<td></td>
<td>24.9 GPa</td>
<td>53.7 GPa</td>
</tr>
<tr>
<td>(E_3)</td>
<td>7.4 GPa</td>
<td>11.7 GPa</td>
</tr>
<tr>
<td>(G_{13})</td>
<td>1.9 GPa</td>
<td>4 GPa</td>
</tr>
<tr>
<td>(\nu_{12})</td>
<td>0.15</td>
<td>0.31</td>
</tr>
<tr>
<td>(\tau_{13})</td>
<td>49 MPa</td>
<td>79 MPa</td>
</tr>
<tr>
<td>(\varepsilon_f)</td>
<td>0.021</td>
<td>0.0138</td>
</tr>
<tr>
<td>(\tau_{IRSS})</td>
<td>13 MPa</td>
<td>50 MPa</td>
</tr>
<tr>
<td>(e_t)</td>
<td>4.98 MJ/m^3</td>
<td>5.1 MJ/m^3</td>
</tr>
<tr>
<td>(G_{\text{IC}})</td>
<td>2.8 KJ/m^2</td>
<td>0.8 KJ/m^2</td>
</tr>
<tr>
<td>(B)</td>
<td>1.64</td>
<td>0</td>
</tr>
<tr>
<td>(\sigma_e)</td>
<td>250 MPa</td>
<td>85 MPa</td>
</tr>
</tbody>
</table>
As shown in Figure 3 and Figure 4, local failure model caused by wave are in good agreement with experimental data; however local deformation model, couldn't predict ballistic limit properly. With any increment in plate thickness, difference between experimental results and prediction of local deformation model become more noticeable.

Figure 3: Comparison between ballistic limit velocity obtained from represented model and experimental data in reference [4]

Figure 4: Comparison between ballistic limit velocity obtained from represented model and experimental data in reference [5]
In the next step effect of plate dimensions and projectile diameter on ballistic limit was investigated. Figure 5 shows variation in ballistic limit velocity with a carbon/epoxy E-Glass/vinylester plate dimensions. Laminate plates thickness is 2.5 mm which subjected to 14 gr and 12.7 mm diameter flat head projectile. By considering Figure 5 it becomes obvious that in thin plates when dimension increase ballistic limit velocity be higher.

![Figure 5: Plate dimension effect on ballistic limit velocity for E-glass/vinylester](image)

Figure 5: Plate dimension effect on ballistic limit velocity for E-glass/vinylester

Figure 6 shows ballistic limit variation to projectile diameter for 3.2 mm thick epoxy/carbon laminate. Projectile mass is 14 gr which its length varies with diameter variation. With any increment in projectile diameter predicted ballistic limit velocity will increase.

![Figure 6: Ballistic limit velocity varies with changes in plate thickness to projectile diameter ratio for epoxy/carbon.](image)

Figure 6: Ballistic limit velocity varies with changes in plate thickness to projectile diameter ratio for epoxy/carbon.
As the analytical model is in good agreement with experimental results, ballistic velocity for materials listed in Table 2 is now calculated and be compared with software results.

Table 2: Projectile mechanical characteristics

<table>
<thead>
<tr>
<th>$\rho$ (Kg/m$^3$)</th>
<th>$E$ (GPa)</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7830</td>
<td>207</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 3, 4, 5 and 6 represent Epoxy-Carbon Elastic properties, E-glass/epoxy Elastic properties, Epoxy-Carbon strength properties and Analysis E-glass/epoxy strength properties which were used in ANSYS/LS-DYNA modelling setup.

Table 3: Epoxy-carbon elastic properties in software modeling setup

<table>
<thead>
<tr>
<th>$E_{11}$ (GPa)</th>
<th>$E_{22}$ (GPa)</th>
<th>$E_{33}$ (GPa)</th>
<th>$E_{12}$ (GPa)</th>
<th>$E_{13}$ (GPa)</th>
<th>$E_{23}$ (GPa)</th>
<th>$V_{12}$</th>
<th>$V_{13}$</th>
<th>$V_{23}$</th>
<th>$\rho$ (Kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>52.1</td>
<td>52.1</td>
<td>8</td>
<td>3.89</td>
<td>3.8</td>
<td>3.8</td>
<td>0.045</td>
<td>0.064</td>
<td>0.064</td>
<td>1306</td>
</tr>
</tbody>
</table>

Table 4: E-glass/epoxy elastic properties in software analysis

<table>
<thead>
<tr>
<th>$E_{11}$ (GPa)</th>
<th>$E_{22}$ (GPa)</th>
<th>$E_{33}$ (GPa)</th>
<th>$E_{12}$ (GPa)</th>
<th>$E_{13}$ (GPa)</th>
<th>$E_{23}$ (GPa)</th>
<th>$V_{12}$</th>
<th>$V_{13}$</th>
<th>$V_{23}$</th>
<th>$\rho$ (Kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.15</td>
<td>21.16</td>
<td>8.3</td>
<td>3.99</td>
<td>4.1</td>
<td>4.1</td>
<td>0.169</td>
<td>0.270</td>
<td>0.276</td>
<td>1750</td>
</tr>
</tbody>
</table>

Table 5: Epoxy-carbon strength properties in software analysis

<table>
<thead>
<tr>
<th>$X_T$ (MPa)</th>
<th>$Y_T$ (MPa)</th>
<th>$Z_T$ (MPa)</th>
<th>$S_{12}$ (MPa)</th>
<th>$S_{13}$ (MPa)</th>
<th>$S_{23}$ (MPa)</th>
<th>$X_C$ (MPa)</th>
<th>$Y_C$ (MPa)</th>
<th>$Z_C$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>545</td>
<td>545</td>
<td>48.5</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>410</td>
<td>410</td>
<td>210</td>
</tr>
</tbody>
</table>

Table 6: E-glass/epoxy strength properties in software analysis

<table>
<thead>
<tr>
<th>$X_T$ (MPa)</th>
<th>$Y_T$ (MPa)</th>
<th>$Z_T$ (MPa)</th>
<th>$S_{12}$ (MPa)</th>
<th>$S_{13}$ (MPa)</th>
<th>$S_{23}$ (MPa)</th>
<th>$X_C$ (MPa)</th>
<th>$Y_C$ (MPa)</th>
<th>$Z_C$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>213</td>
<td>213</td>
<td>26.2</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>220</td>
<td>220</td>
<td>130</td>
</tr>
</tbody>
</table>

29
The model which was created in ANSYS/LS-DYNA is bilinear isotropic solid with SHELL181 element. Elements contact are TARGE170 and CONTA173 with friction coefficient MU=0.3. Figures 7, 8 and 9 represent finite element modelling in ANSYS/LS-DYNA software.

Figure 7: Projectile and epoxy-carbon composite with thickness of 3.2 mm perforated by projectile

Figure 8: Projectile and 4.5 mm E-glass/epoxy composite laminates before strike

Figure 9: 4.5 mm thick E-glass/epoxy laminate perforated by a projectile
As it is shown in Figure 10 and Figure 11 there is a little difference between analytical results and software simulation results. The main reason for such a difference comes from not considering Delamination Energy in software solution. Laminates has been modeled by ANSYS/LS-DYNA. The results show that with incretion in laminate thickness, ballistic limit velocity will also increase.
Conclusion

In this paper, ballistic limit velocity and perforation in fibre reinforced plastic laminates subjected to strike were investigated. Two main category of failure have been studies; the first one is complete deformation with centralized failure and the second one is local failure which was caused by wave. In presenting analytical model for complete deformation, semi-statical method have been employed to estimate the absorbed energy. Moreover; mechanical increment factor has been used for calculating perforation energy. By combining Von Karman critical velocity method with local failure model for governing wave, the second failure mode was gained. Theoretical results for ballistic limit wave prediction are in good agreement with experimental results. Presented analytical model is simple and agrees with experimental results. By means of proposed model it could be concluded that when plate dimensions and projectile diameter increase ballistic limit velocity will also increase. Future research should focus on different shapes of projectile when encountering a target with different kind of materials.

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