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The Prediction of Transmission Loss Using Transfer Matrix Method

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ABSTRACT

Mufflers play an important role in attenuating the noise level produced by noise source especially in automotive application. A reliable, effective and affordable method in assessing and determining muffler performance in term of sound absorption criteria is therefore an important aspect to study. Different muffler configurations such as inlet and outlet pipe diameter, baffle locations, chamber length are studied with respect to the transmission loss (TL) in wide frequency range. Analytical analysis based on Transfer Matrix Method (TMM) was utilised and source code using FORTRAN was developed to accurately predict the transmission loss. The predicted numerical results with selected parameters show a very good agreement with results obtained experimentally and BEM.

Keywords: *Transfer matrix, transmission loss, muffler design*

Introduction

Mufflers are part of exhaust system commonly installed along the exhaust pipe to reduce noise level while maintaining uninterrupted flow of the exhaust gas. Commonly used in automotive, mufflers usage is demanded by authority to attenuate the noise levels radiated to the atmosphere and to certain extent absorb some of the harmful hazardous gases[1]. Noise not only creates unpleasantness, but also adversely affects the system performance through energy losses. The automobile generated noise is one of the most predominant sources of annoyance,

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particularly in urban areas [2]. Since muffler is the main contributor of noise suppression in an exhaust system, therefore reliable muffler design is pertinent to the whole successful function of the exhaust system.

There are several procedures and methods that are available for analysing noise reduction performance of mufflers namely empirical, analytical and numerical methods [3-5]. The method used dependent highly on the particular needs for the analysis [6-9]. Recent improvement in modelling procedures and methods for accurate performance prediction have led to the development of modelling methods for practical muffler components in commercial design [9, 10]. Numerical techniques, such as the Finite Element Method (FEM), Computational Fluids Dynamic (CFD) and the Boundary Element Method (BEM) have been widely applied [5, 11-13] have proven to be effective for design of muffler with complex geometries. Although these techniques are good on designing and analysing any muffler configurations, with frequency variations and complex muffler shape, the FEM model needs large computing power and speed plus trained personnel is required to model the system effectively. Apart from that, the commercially available software is relatively expensive.

The transfer matrix method may be an efficient tool to deal with branched acoustic systems. The overall transfer matrix of the whole acoustic system can be obtained by assembling all the transfer matrices of the sub-domains[14]. The method has limitation in application since it is based on the assumption of plane wave propagation at the interface of sub-domains, which is only true below the cut off frequency. The overall transfer matrix of the silencer is calculated through serial multiplication of the sub-domain transfer matrices. In this study the development of FORTRAN based soft code utilising Transfer Matrix Method (TMM) is discussed to predict transmission loss (TL) of different muffler configurations for design purpose circular shape muffler. However, the interaction of the heat and fluid dynamics is not included in the analysis. The efficiency and capabilities of the code is initially validated with published experimental data [6, 12], before variations of the parameters are carried out.

Mathematical Formulation

To formulate the transmission loss equation, a plane wave propagation theory in a rigid straight pipe is considered. Assuming that of the pipe length L, the constant cross section S, the transporting a turbulent incompressible mean flow of velocity V (see Figure 1), the acoustic pressure perturbation (on the ambient static pressure) p and the particle velocity u, therefore anywhere in the pipe element can be represented as the sum of left and right traveling waves. The plane wave propagation model is valid when the influence of higher order modes can be neglected.

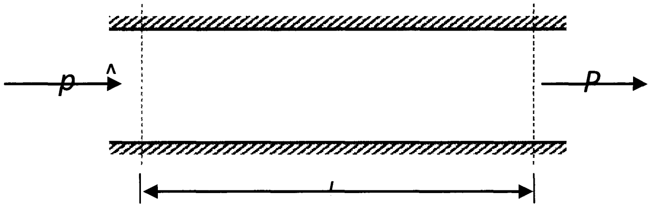

Figure 1: Plane Wave Propagation in a Rigid Straight Pipe Transporting a Turbulent Incompressible Mean Flow

Using the impedance analogy, the sound pressure p and volume velocity v at position *1* (upstream end) and *2* (downstream end) in Figure 1 ($x = 0$ and $x = L$, respectively) can be related by

$$p_1 = T_{11}p_2 + T_{12}v_2, \tag{1}$$

and

$$v_1 = T_{21}p_2 + T_{22}v_2, \tag{2}$$

where T_{11}, T_{12}, T_{13}, and T_{14} are called the four-pole constants. They are frequency-dependent complex quantities embodying the acoustical properties of the pipe [15].

It can be shown [14] that the four-pole constants for non viscous medium are

$$T_{11} = \exp(-jMkc\, L) \cos kc\, L, \tag{3}$$

$$T_{12} = j(\rho\, c\, /\, S) \exp(-jMkc\, L) \sin kc\, L, \tag{4}$$

$$T_{21} = j(S\, /\, \rho\, c) \exp(-jMkc\, L) \sin kc\, L, \tag{5}$$

$$T_{22} = \exp(-jMkc\, L) \cos kc\, L, \tag{6}$$

where $M = V/c$ is the mean flow Mach number ($M < 0.2$), c is the speed of sound (m/s), $kc = k/(1 - M_2)$ is the convective wave number (rad/m), $k = \omega/c$ is the acoustic wave number (rad/m), ω is the angular frequency (rad/s), ρ is the fluid density (kg/m^3), and j is the square root of -1.

Transfer Matrix Method

Adopting acoustic pressure p and mass velocity as v as two state variables, the following matrix relation can be written so as to relate state variable on two sides of the element subscripted r in the equivalent form;

$$\begin{bmatrix} p_r \\ v_r \end{bmatrix} = \begin{bmatrix} T & 2\times 2\ Transfer\ Matrix \\ & for\ the\ r_h\ \ element \end{bmatrix} \begin{bmatrix} p_{r-1} \\ v_{r-1} \end{bmatrix} \tag{7}$$

where, $[p_r, v_r]$ is the state vector at the upstream point r and $[p_{r-1}, v_{r-1}]$ is the state vector at the downstream point $r-1$. The transfer matrix for the r_{th} element can be denoted by $[T_r]$ and can be written as,

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \tag{8}$$

where,

$$T_{11} = \frac{p_r}{p_{r-1}}\bigg|_{v_{r-1=0}} \qquad T_{12} = \frac{p_r}{v_{r-1}}\bigg|_{p_{r-1=0}}$$
$$T_{21} = \frac{v_r}{p_{r-1}}\bigg|_{v_{r-1=0}} \qquad T_{22} = \frac{v_r}{v_{r-1}}\bigg|_{p_{r-1=0}} \tag{9}$$

The elements of the transfer matrix, T_{11}, T_{12}, T_{21} and T_{22} represent individual physical significance, where;

T_{11} = Ratio of the upstream pressure and down stream pressure end
T_{21} = Ratio of the upstream velocity to the pressure at the downstream end
T_{21} = Ratio of the upstream velocity to the pressure at the downstream end
T_{22} = Ratio of the upstream velocity and down stream velocity end

Thus, the transmission loss of a muffler can be obtained from elements of the overall transfer matrix of element [16],

$$\begin{bmatrix} p_n \\ v_n \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} \tag{10}$$

The overall transfer matrix relation can then be written by successive application of definition given in as

$$\{S_n\} = [T_r]..[T_{r+1}]...\{S_o\} \tag{11}$$

where;

$[T_r]$ = Product transfer matrix.
$\{S_n\}$ = The state vector at n station

From Equation (1), we write

$$p(z,t) = \left[C_1 e^{-jkz} + C_2 e^{+jkz}\right] e^{j\omega t}$$

By simplifying Equation (1), it becomes

$$P_n = A_n + B_n \tag{12}$$

From Equation (2), we write

$$v(z,t) = \frac{1}{Y_o}(C_1 e^{-jkz} - C_2 e^{+jkz})$$

By simplifying Equation (2)

$$V_n = \frac{(A_n - B_n)}{Y_n} \tag{13}$$

where,

$$P_1 = A_1 + B_1 = A_1 \text{ (as } B_1 = 0) \tag{14}$$

$$V_1 = (A_1 + B_1)/Y_1 = A_1/Y_1 \tag{15}$$

$$A_n = \frac{(P_n + Y_n v_n)}{2} \tag{16}$$

Transmission loss, TL, in dB is then defined as,

$$TL(dB) = 20\log_{10}\left|\frac{A_n}{A_1}\right| \tag{17}$$

Substituting and rearranging equations (12), through (16) into Equation (17), we have

$$TL(dB) = 20\log_{10}\left|\frac{P_n + Y_n v_n}{2Y_1 v_1}\right| \tag{18}$$

$$A_n = \frac{\left[\left(T_{11}A_1 + T_{12}\frac{A_1}{Y_1}\right) + Y_n\left(T_{12}A_1 + T_{22}\frac{A_1}{Y_1}\right)\right]}{2} \tag{19}$$

Equation (19) yield,

$$\frac{A_n}{A_1} = \frac{1}{2}\left[T_{11} + \frac{T_{12}}{Y_1} + Y_n T_{21} + \frac{Y_n}{Y_1}T_{22}\right] \tag{20}$$

The transmission loss is then calculated using the transmission matrices and can be expressed as follows.

$$TL = 20\log\left[\left(\frac{Y_1}{Y_n}\right)^{\frac{1}{2}}\left[\frac{T_{11} + \frac{T_{12}}{Y_1} + Y_n T_{21} + \left(\frac{Y_n}{Y_1}\right)T_{22}}{2}\right]\right] \quad (21)$$

where, Y is the longitude wave which is the characteristics impedance of the pipe and and is defined as a ratio, $Y = a_0/S$. Here, Y_1 and Y_n represent individual sound impedance from outlet-inlet pipe of the muffler, respectively.

Alternatively, transmission Loss may be defined the power incident level on the muffler proper (Lw_i) and that transmitted downstream (Lw_t) [1] as follow:

$$TL = Lw_i - Lw_t \quad (22)$$

$$TL(dB) = 20\log_{10}\left|\frac{w_i}{w_t}\right| = 20\log_{10}\left|\frac{P_1^+}{P_2^+}\right| = 20\log_{10}\left|\frac{A_n}{A_1}\right| \quad (23)$$

In the present study, the wave equation for one dimensional exhaust system with an element of muffler chamber as shown in Figure 2 is calculated using principle of mass flux of the flow properties.

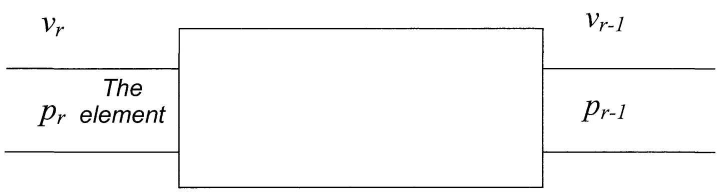

Figure 2: The General Representation of An Element of Muffler Chamber

The preceding expressions for the Transfer Matrix Method enable one to measure each of the parameters for an element or subsystem of the muffler.

Numerical Model

The engine exhaust system in the automobile is divided into four components, namely, engine, exhaust pipe, muffler and tail pipe which are assumed to be infinitely anechoically terminated so that there is only one wave in the outlet of the pipe as in Figure 2. The expansion from inlet pipe results in a reflected plane wave propagating away from the muffler. The transfer matrix constants of each element are not affected by the connection of elements upstream or downstream. Furthermore, Eq. (8) is only applicable for mufflers that are considered lumped parameter and one-dimensional acoustics elements system. So each element is characterized by one transfer matrix as shown in Figure 3.

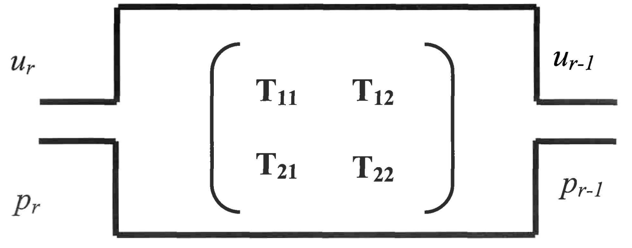

Figure 3: Transfer Matrix Method Model

It is necessary to model each element and then to relate each of them to obtain the overall acoustic property of the muffler. Based on the muffler modelled as in Figure 3, the computational program is developed and is then utilised to analyze the acoustic performance of muffler. In this case a one dimensional (1-D) wave is assumed. Further to that the same developed numerical program based on the Transfer Matrix Method is applied to more complex configurations as illustrated in Figure 4.

The numerical analysis is carried out with several different configurations such as variation of area ratio between inlet pipe and muffler chamber, variation of muffler chamber diameter, variation of muffler chamber length, variation of inlet temperature, variation of baffle location and variation of exhaust outlet pipe diameter. In each case the transmission loss is calculated and analysed.

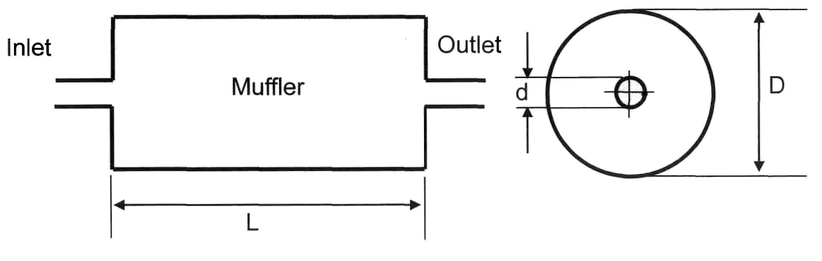

Figure 4: The Configurations of Muffler Area Ratio between Inlet Pipe, Outlet Pipe and Muffler Chamber

Results and Discussions

Figure 5 shows the results of the transmission loss obtained from the Transfer Matrix Method. The frequency range used is 0 Hz to 3000 Hz, similar to existing literature. The first and second humps are close to 20 dB while the third hump is around 22 dB. However a drastic drop in transmission loss occurred at the last hump i.e. above 2500 Hz.

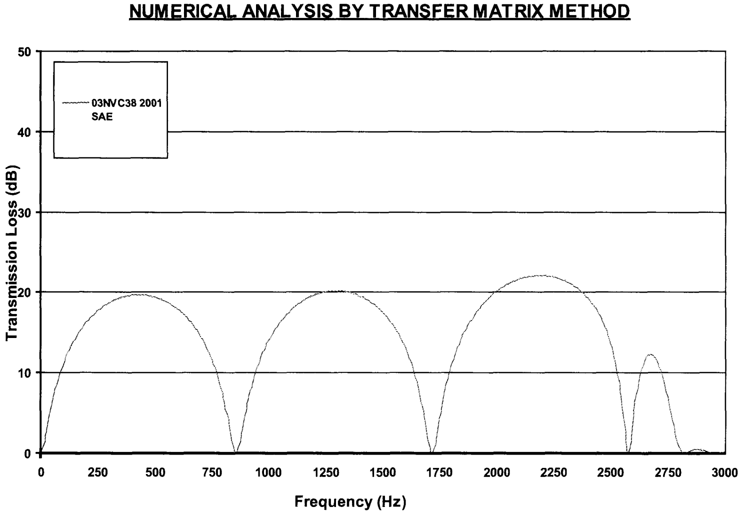

Figure 5: Transmission Loss Calculated Using Transfer Matrix Method

Figure 6 depict the close agreement of the result of the proposed method, the empirical (measured) and the numerical (boundary element method) using the inlet and the outlet muffler as 1.375 inch (34.925 mm) respectively and the diameter of muffler chamber is 6.035 inch (153.2 mm). As can be observed, the numerical results show a very good agreement with the empirical results within the frequencies range of 0 Hz and 2500 Hz.

Again, the magnitude and pattern of the hump demonstrated by the three methods match very closely, although the difference tends to be more pronounce with an increase in the frequency. The transmission loss starts becoming an unpredicted as the frequency is above 2500 Hz and as before this can be seen at the last hump.

The effect of area ratio between inlet pipe and muffler chamber is presented in graphical form, Figure 7. Five cases of different area ratio with different magnitudes and patterns of hump are shown. The magnitude and pattern of hump which represents the transmission loss of muffler clearly depends on the area

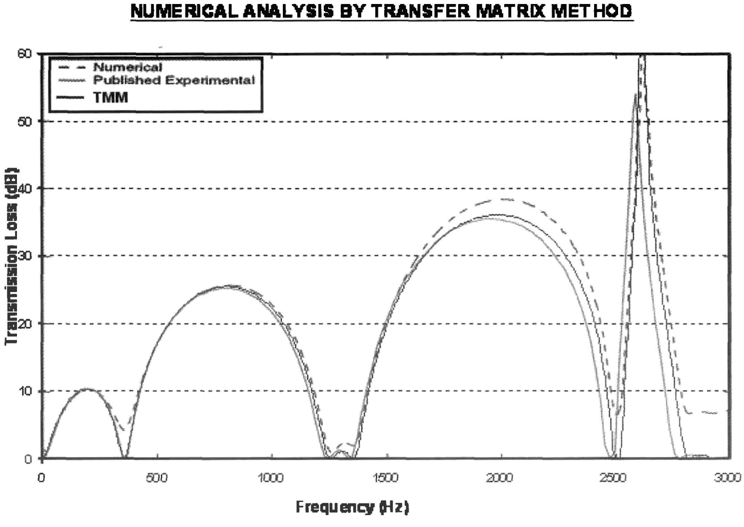

Figure 6: Transmission Loss of Muffler with Baffle Using the Developed TMM and It Comparison with Other Established Methods

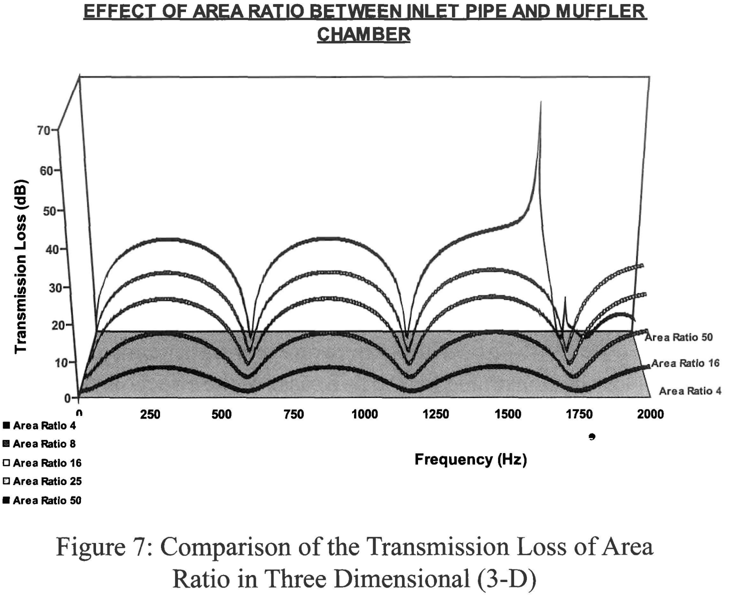

Figure 7: Comparison of the Transmission Loss of Area Ratio in Three Dimensional (3-D)

ratio considered. Increasing the area ratio will increase the transmission loss and also will scale up the hump pattern. In this case, the lowest hump pattern is demonstrated by case 1 with area ratio of 4 while the highest one is for case 5 with area ratio of 50.

Figure 8 shows the five cases of different muffler diameter with different magnitudes and patterns of hump which represent the transmission loss. The magnitude and pattern of hump clearly depends on the muffler diameter considered. Increasing the muffler diameter increases the transmission loss and also scale up the hump pattern. It is observed that, the lowest pattern of hump is demonstrated by the diameter of 150 mm while the highest one is for the diameter of 350 mm.

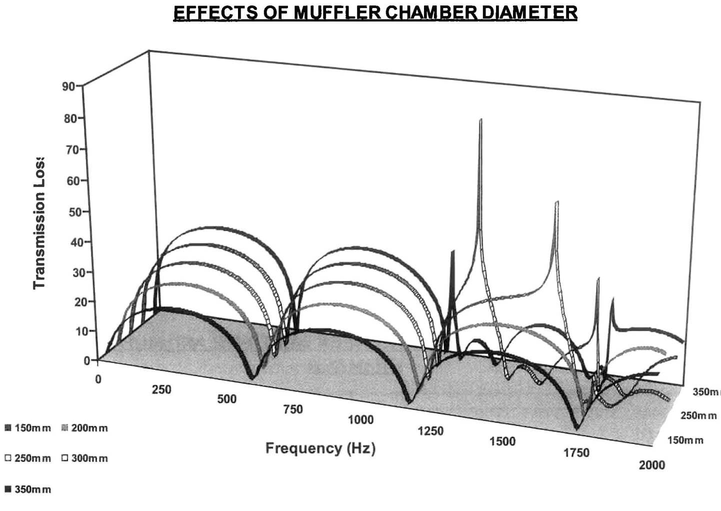

Figure 8: Comparison of the Transmission Loss of Muffler Diameter in Three Dimensional (3-D)

From Figure 9, it shows that there are no significant changes of the transmission loss demonstrated for the five cases of the effect of muffler chamber length. However the magnitude and pattern of the humps are changing as the muffler chamber length is increased. The longer the muffler chamber length, the more the number of humps and also the smaller the magnitude of humps.

From Figure 9a, it shows that there are no significant changes of the transmission loss in the five cases of the effect of inlet temperature. The significant changes are merely presented by the magnitude of hump. The higher the inlet temperature, the bigger the magnitude of humps and the number of humps become lesser. However, the inlet temperature does not affect the transmission loss of the muffler.

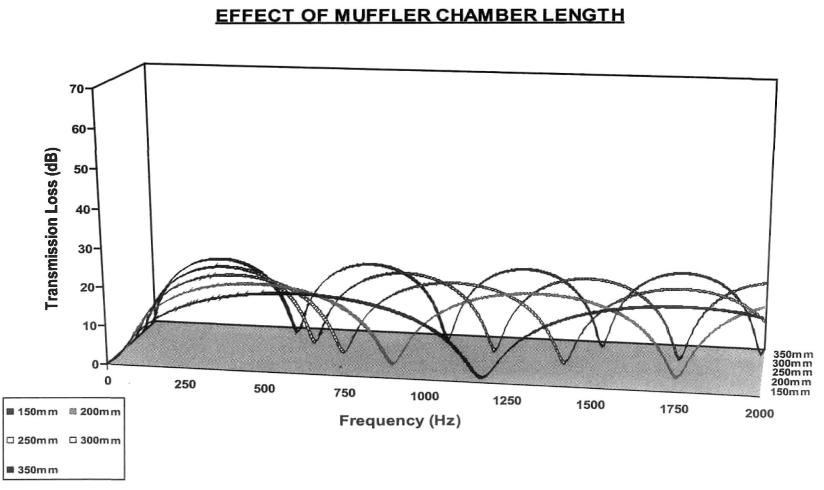

Figure 9: Comparison of the Transmission Loss of Muffler Chamber Length in Three Dimensional (3-D)

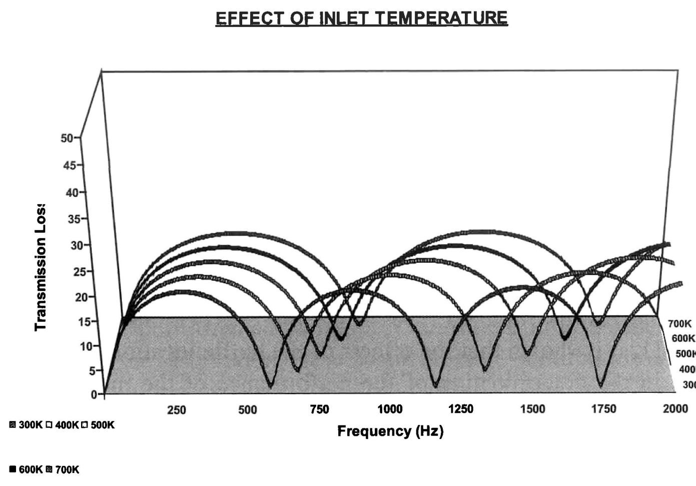

Figure 9a: Comparison of the Transmission Loss of Inlet Temperature in Three Dimensional (3-D)

The effect of varying the location of baffle is shown in Figure 10. The results show that the further distance of baffle from inlet pipe, the better the performance of transmission loss. The best performance is obtained at 100 mm from pipe inlet of muffler chamber. However the three dimensional waves

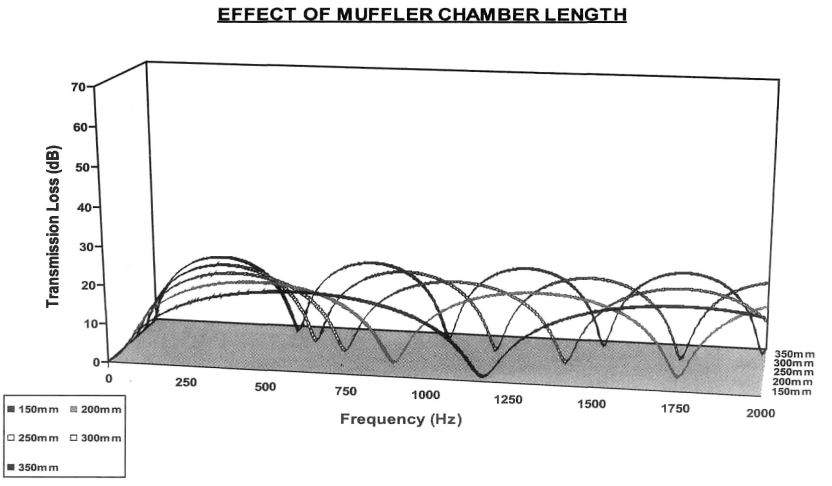

Figure 9: Comparison of the Transmission Loss of Muffler Chamber Length in Three Dimensional (3-D)

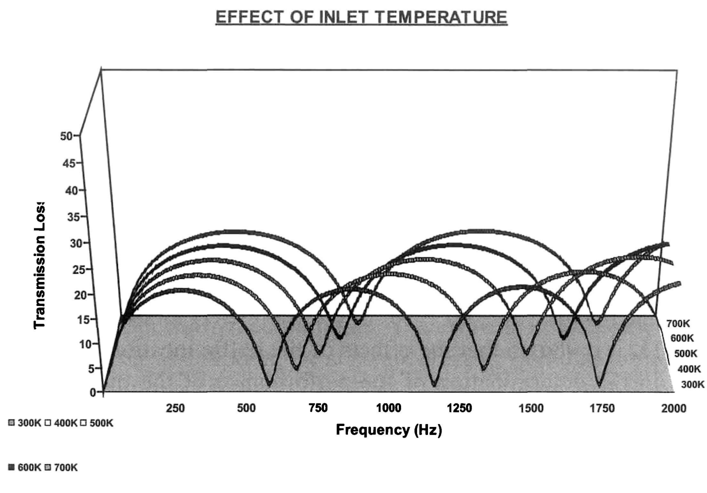

Figure 9a: Comparison of the Transmission Loss of Inlet Temperature in Three Dimensional (3-D)

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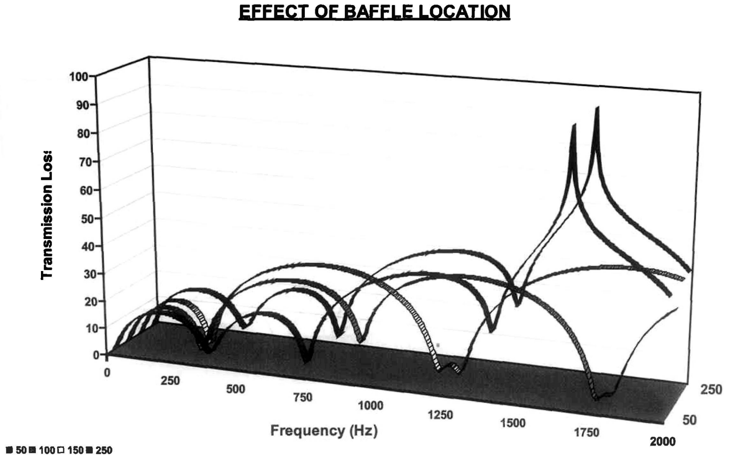

Figure 10: Comparison of the Baffle Location in Three Dimensional (3-D)

occur when the location of the baffle is nearest and furthest to inlet and outlet pipe, respectively.

Figure 11 shows the five cases of different muffler diameter with different magnitudes and patterns of hump which represent the transmission loss. The magnitude and pattern of hump depends on the muffler diameter considered. It is observed that, increasing the muffler diameter decreases the transmission loss and hump pattern. However at the frequency above 1170 Hz, the transmission loss patterns demonstrated as turbulence wave. In this case, the lowest pattern of hump is demonstrated by the diameter of 100 mm while the highest one is for the diameter of 60 mm.

From the results of the six types of the analyses demonstrated in Figures 7 to Figure 11, it is shown that the effects of the baffle location have the largest influence in the determination of the performance of the muffler expansion chamber. This is due to the baffle in the chamber produces a relatively high pressure drop at the exit of baffle location.

The second most influential factor in determining the transmission loss is the effects of area ratio between pipe and muffler diameter Figure 17, and followed by the third one is the effects of muffler chamber diameter Figure 9. Meanwhile the fourth influential factor is the effects of outlet pipe diameter Figure 11. This is due to in practice, the expansion of area ratio is defined as the area of the muffler to the inlet pipe that controls the amplitude of the expansion chamber humps as shown in Figure 7 and Figure 8. Transmission loss can be increased by either by increasing the body diameter of muffler chamber or decreasing the inlet pipe diameter of the muffler. However reducing the inlet pipe diameter is not

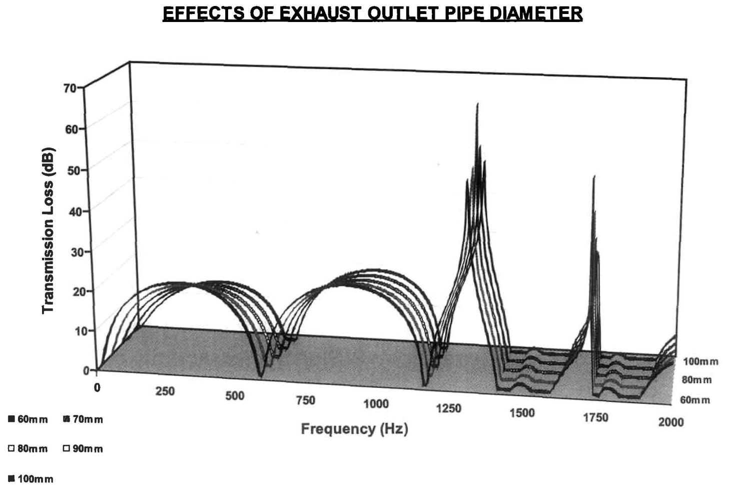

Figure 11: Comparison of the Transmission Loss of Outlet Pipe diameter.

desirable and increasing the body diameter of muffler chamber is the only viable solution to increase transmission loss results as demonstrated in Figure 8.

Conclusion

This study was successfully carried out as an alternative method for analysing the noise performance of an automobile exhaust muffler using Transfer Matrix Method based on linear plane wave propagation theory and numerically solved with FORTRAN source code. This study utilised the Transfer Matrix Method to analyse muffler performance particularly transmission loss based on plane wave theory with the assumption, the acoustic pressure perturbation and particle velocity at all point of a cross section in the duct was the same. Meanwhile the medium was considered as homogeneous, constant temperature, and laminar flow. However the limitation of this study is restricted to non-perforated type muffler chamber and cylindrical shaped muffler.

Numerical validation was carried out with simple muffler configuration. The transmission losses calculated were in closed agreement with the experimental and numerical method. The method was further developed to analyse the transmission loss of other mufflers with different parameter settings. Six types of analyses with different parameter settings which are the variations effects of the area ratio between pipe and muffler diameter, variations effects of muffler chamber diameter, variations effects of muffler chamber length, variations effects of inlet temperature, variations effects of baffle location and effects of exhaust pipe diameter were successfully carried out with numerous numerical results of the transmission loss.

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