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Springback Analysis of Thin Tubes with Arbitrary Stress-Strain Curves

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ABSTRACT

A general theoretical method for determining springback of arbitrary shaped thin tubular section of materials having arbitrary stress-strain relationship under torsional loading is presented. The theoretical analysis has been compared with earlier analysis of tubular sections of particular materials and has been shown that the expressions obtained in the, work presented. The theoretical results also have been found to quit in agreement with the results obtained experimentally. It has been shown that when applied torque is kinetic loading or angle of twist given is kinetic loading, springback angle and residual angle of twist can directly

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be calculated from the shear stress-strain curve, avoiding any idealization/ approximation for the same.

Keywords: *Metal forming, Springback, Torsional springback, Thin tubes, Arbitrary stress-strain curves*

Nomenclature

u, v, and w	Small displacements in x, y, and z direction, respectively
θ	Angle of twist per unit length of the tube
θ_R	Residual angle of twist per unit length
$\bar{\theta}$	Non-dimensionalized angle of twist per unit length
$\bar{\theta}_R$	Non-dimensionalized residual angle of twist
γ	Shear strain
τ_{xz}, τ_{yz}	Shear stress
G	Modulus of rigidity
$\emptyset$	Stress function
∇	Gradient
T	Twisting moment (torque)
T_O	Twisting moment at yielding
$\bar{T}$	Non-dimensionalized torque
ξ	Deflection of membrane
σ_x	Normal stress
β	Angle made by the line of length ρ with the tangent to the contour element
q	Constant (shear flow)
ρ	Distance of the element from the origin
A	Mean of the areas enclosed by the outer and inner boundaries of the cross-section of the tube
S	Length of the center line of ring section of the tube
t	Thickness of the tube
τ_o	Yield shear stress
γ_o	Yield shear strain
α	Plastic modulus of rigidity
ε_x, ε_y	Longitudinal strain
μ	Poisson's ratio

Introduction

A large variety of aerospace, automotive, appliances, structural and other industrial components are made from thin walled tubes. Theses tubes may be in different cross-sections such as circular, square, rectangular, triangular etc. Such components are advantages from view-point of reduction in weight alongwith improved structural strength. Tubes of different cross-sections have produced by forming the metal sheets by plastic bending and torsion with the aid of punches and dies. After the end of bending process and on the removal of tooling, the dimensions of bend tube changes due to elastic recovery behavior of tube material. This phenomenon is called springback. During the process of bending the internal stresses are induced in tube but they are not vanishes on unloading. After bending the extrados and intrados are subjected to both type of residual stresses i.e. tensile and compressive respectively. Such residual stresses produce a net internal bending moment which causes springback. Springback is a very important factor which has to be taken into consideration, while designing the tool on loading a body plastically, particularly in sheet metal working. Springback has to be eliminated or corrected at the initial phases of design to achieve consistent and accurate dimensions of the final part. It is affecting the whole product development and manufacturing process in terms of production delay, higher rejection rate and increment in cost for tooling revisions. All these factors reduce customer satisfaction and loyalty for the final product. US automotive industries alone bear a loss of more than $50 million per year [1].

Springback is influenced by number of parameters both from material and process selected. Theses may be material properties, sheet thickness, elastic modulus, Poisson's coefficient, blank material, punch and die radii, initial clearance friction conditions, binder force, tooling geometry, blankholder force, blankholder geometry, and geometry of draw beads [2]. The relationships that exist between springback and these parameters are extremely nonlinear with multiple interactions [3]. Based on the part geometry and deformation area, different types of springback in sheet metal exist: bending, membrane, twisting and combined bending and membrane [4]. The twisting or torsional type of springback is the measure of elastic recovery of angle of twist on the removal of applied torque after twisting the section beyond elastic limit. Uneven elastic recovery in different directions is the main cause for this type.

In the last 40 years, numerous research efforts have been made to reduce springback by effectively predicting and determining the controlling parameters in the sheet bending operations through experiments [5] - [19] and simulation [20] - [27]. In all these works, issues related to accurate prediction and effective compensation of springback for different type geometries are discussed. In recent years, much attention has been placed on springback of tubes [28] - [32]. Most of the researchers use simplified models and different stress-strain relationship in finding out the amount of springback in tube bending operations. Torsional springback of bars of different cross-sections has been analyzed by Dwivedi et al. [33] - [38]. Springback of narrow rectangular strips of linear work hardening materials under torsional loading [33] - [34] and general cross-sections with the torsional springback for work-hardening materials were estimated [35] - [37]

by using Ramberg-Osgood stress-strain relation and deformation theory of plasticity. A numerical scheme based on finite difference approximation was used. The elastic-plastic boundary in the bars of square section and L-section were also determined in addition to springback. Theoretical results were verified by experiments on mild steel bars. In a study [38], springback analysis of narrow rectangular bar in torsion for non-linear work-hardening materials was considered. Non-linear behavior of the material was approximated by assuming Modified Ludwick type stress-strain relationship. The result of bi-linear case [33] was compared with by considering non-linear behavior of the material of tubular bars. It was found that experimental values had excellent match for materials having non-linear behavior. Recently, Choubey et al. [39] developed a theoretical expression for torsional springback in thin tubes with non-linear work hardening behavior of the material and compared them with experimental findings for mild steel case.

The present work is concerned with a theoretical method of determining springback in thin tubes with arbitrary sections of materials having arbitrary stress-strain relationship avoiding any approximation/idealization of the same. It has been shown that springback/residual angle of twist can be obtained from the applied torque in kinetic loading or from the given angle of twist in kinematic loading directly using the shear stress-strain curve. Bi-linear stress-strain relationship has been considered as a particular case of zone general behavior and the expressions obtained by using bi-linear relationship in the general analysis have been found to be exactly the same expressions obtained earlier in reference [36].

Basic Theory

Consider a prismatic thin tube undergoing torsion and elastic deformation (Figure 1). Let u, v and w be the small displacements of the point (x, y, z) relative to its initial position, in the x, y and z directions respectively. At a section, where z is constant, the cross-section rotates about the z-axis and so [38], [39].

$$u = -yz\theta, \qquad v = xz\theta, \qquad \text{and} \qquad w = \theta f(x,y) \tag{1}$$

where is θ the angle of twist per unit length of the tube.

Let it be assumed that even in plastic range considered in this article the displacement of a point (u, v, w) are small enough so that the strain displacement relationships are in the elastic range.

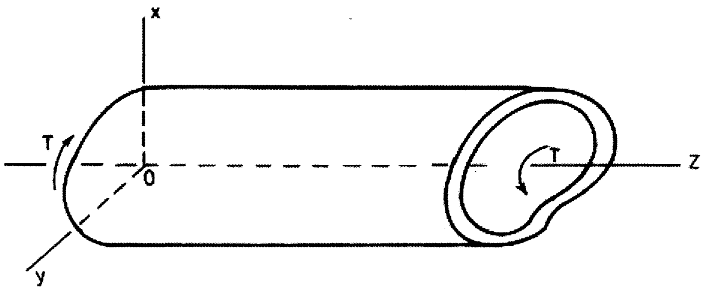

Figure 1: Geometry of the Problem

$$\epsilon_x = \epsilon_y = \epsilon_x = \emptyset$$

$$\gamma_{xy} = \frac{\partial W}{\partial x} + \frac{\partial u}{\partial z} = \theta\left[\frac{\partial \psi}{\partial x} - y\right] \tag{2}$$

$$\gamma_{yz} = \frac{\partial W}{\partial y} + \frac{\partial v}{\partial z} = \theta\left[\frac{\partial \psi}{\partial y} + x\right]$$

are valid in the plastic range also.

Let it be assumed that the material of the tube has the arbitrary stress strain relationship given by

$$\tau = f(\gamma) \tag{3}$$

which degenerates into the relationship

$$\tau = G\gamma \tag{4}$$

in the elastic range. G being the modulus of rigidity, from equations (2) and (4)

$$\frac{\partial \tau_{xz}}{\partial y} - \frac{\partial \tau_{yz}}{\partial x} = -2G\theta \tag{5}$$

If a stress function $\emptyset$ is taken such that

$$\tau_{xz} = \frac{\partial \emptyset}{\partial y} \text{ and } \tau_{yz} = -\frac{\partial \emptyset}{\partial x} \tag{6}$$

the equilibrium equation is given by

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \emptyset \tag{7}$$

is automatically satisfied. Again from equation (6) the resultant shear stress is given by

$$\tau = \sqrt{\tau_{xz}^2 + \tau_{yz}^2} = |\text{grad } \emptyset| \tag{8}$$

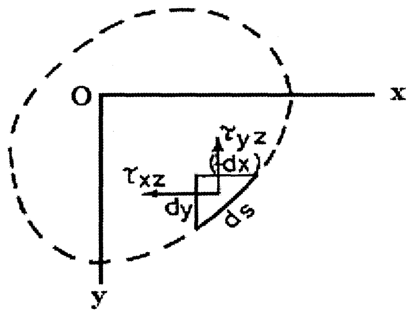

Figure 2: Stresses on an Infinitesimal Element at the Boundary

From equations (5) and (6)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \emptyset = \nabla^2 \emptyset = -2G\theta \tag{9}$$

Since the resultant shear stress τ at a boundary is along the tangent to the boundary (Figure 2)

$$-\tau_{yz}\,dx + \tau_{xz}\,dy = \frac{\partial \emptyset}{\partial x} dx + \frac{\partial \emptyset}{\partial y} dy = d\emptyset = \emptyset \tag{10}$$

i.e. $\emptyset$ = a constant along a boundary. So the difference $\Delta\emptyset$ of $\emptyset$ between the two boundaries of the tube is constant while going along the boundary i. e.

$$\emptyset \approx |\text{ grad } \emptyset\ | \ t = q, \quad \text{a constant} \tag{11}$$

where t is the thickness of the tube at the point considered. So from equation (8) and (11)

$$\tau.\ t = \text{constant} \tag{12}$$

where τ is the average shear stress and q is constant along the mean boundary is called shear flow.

Again considering an elementary mean contour length ds of the section (Figure 3), the tangential force on the element is qds and its moment about any point O is

$$dT = q\ ds\ \rho \sin\beta \tag{13}$$

where ρ is the distance of the point O from the element and β is the angle made by the line of length ρ with the tangent to the contour element [40]. For equilibrium of the tube the applied and tractive torques must be equal i.e.,

$$T = \int_s q\,\rho\,\sin\beta\ ds = 2q\int_s \frac{1}{2}\rho\,\sin\beta\ ds \tag{14}$$

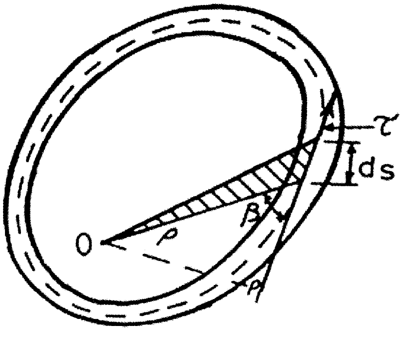

Figure 3: Resisting Torque on the Cross-section of a Thin Tube

But ½ρ sin β ds is the area of the shaded triangle in Figure 3. So,

$$T = 2qA = 2At\,\tau \tag{15}$$

or

$$\frac{T}{2At} = \tau = G\gamma \tag{16}$$

Where A is the area enclosed by the mean boundary.

$$\int \tau \, ds = \int \frac{T}{2At} \, ds = 2G\theta A$$

or

$$\theta = \frac{T}{4A^2G} \int \frac{ds}{t} = \frac{\gamma\, t}{2A} \int \frac{ds}{t}$$

or

$$\gamma = \frac{2A}{t \int \frac{ds}{t}} \theta \tag{17}$$

When the stresses induced by torsion is in the plastic range, from equations (9) and (15)

$$\frac{T}{2At} = f(\gamma) \tag{18}$$

Again since the relationship (17) is independent of the material properties and since it has been assumed that the small strain relationships (2) among strains and displacements are valid also in the plastic range relationship (7) remains valid for plastic torsion. So, from equations (17) and (18)

$$\frac{T}{2At} = f\left(\frac{2A}{t \int \frac{ds}{t}} \theta\right) \tag{19}$$

which shows that the functional relationship between $\frac{T}{2At}$ and $\frac{2A}{t\int\frac{ds}{t}}\theta$ is same as that between shear stress τ and shear strain as shown in Figure 4.

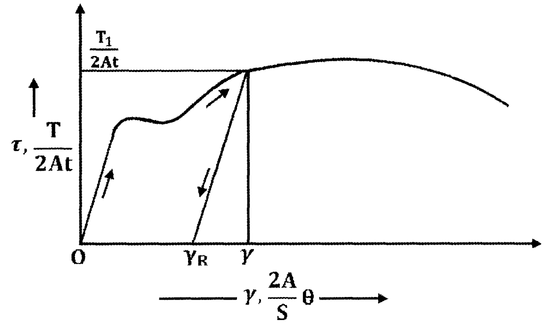

Figure 4: Shear Stress-strain Curve

Hence for a given torque T, angle of twist θ and the residual angle of twist θ_R can directly be obtained from the shear stress-strain curve. When the thickness t of the tube is constant equation (19) becomes

$$\frac{T}{2At} = f\left(\frac{2A}{s}\theta\right) \quad \text{or} \quad T = 2At\ f\left(\frac{2A}{s}\theta\right) \tag{20}$$

However, if from the relationship (3) γ can be explicitly expressed as

$$\gamma = F(\tau) \tag{21}$$

then assuming that unloading is elastic (Figure 5),

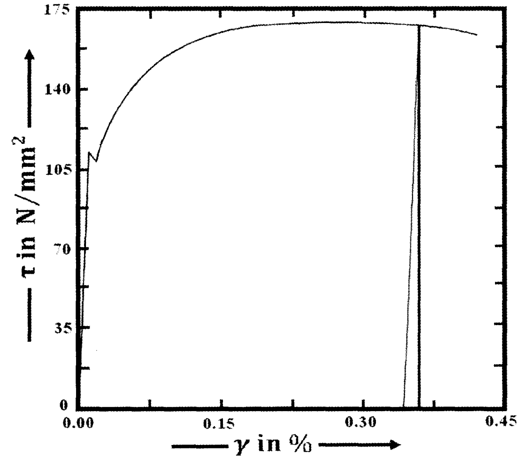

Figure 5: Shear Stress-strain Curve with Elastic Unloading

$$\gamma_R = \gamma - \gamma_e = F(\tau) - \frac{\tau}{G}$$

so,

$$\theta_R = \frac{S}{2A} F\left(\frac{T}{2At}\right) - \frac{T}{4A^2tG} \tag{22}$$

also,

$$\theta_R = \theta - \frac{S f[(2A/s)\theta]}{2AG} F\left(\frac{T}{2At}\right) - \frac{T}{4A^2tG} \tag{23}$$

In case of bi-linear relationship (Figure 6) given by [30]

$$\tau = G\gamma, \qquad \gamma \le \gamma_0$$

$$\tau = \tau_0\left[1 - \frac{\alpha}{G}\right] + \alpha\gamma, \qquad \gamma \ge \gamma_0 \tag{24}$$

Where, τ_0 and γ_0 are yield shear stress and strain respectively and α is the plastic modulus of rigidity given by the slope of the stress strain for $\tau \ge \tau_0$. From the above, γ may be expressed explicitly as

So, τ corresponding to T, $(T > T_0)$is given by

$$\gamma = \alpha/G \qquad \gamma \le \gamma_0$$

$$\gamma = \frac{1}{\alpha}\left[\tau - \tau_0\left(1 - \frac{\alpha}{G}\right)\right] \tag{25}$$

From equations (20). (22) and (23). T and θ_R may be expressed as

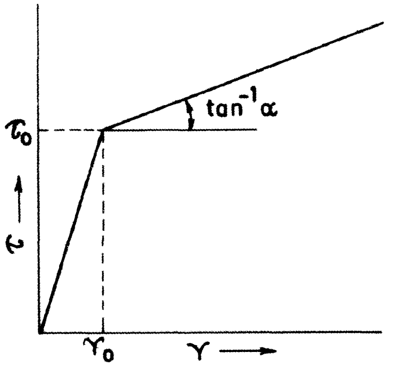

Figure 6: Bi-linear Stress-strain Curve

$$T = T_0\left[1-\frac{\alpha}{G}\right]+\frac{4A^2 t\,\alpha}{s}\theta$$

$$\theta_R = \frac{(T-T_0)s}{4A^2\,t}\left(\frac{1}{\alpha}-\frac{1}{G}\right) \tag{26}$$

$$\theta_R = (\theta-\theta_0)\left[1-\frac{\alpha}{G}\right]$$

writing,

$$\bar{\theta} = \frac{\theta}{\theta_0} = \frac{\theta}{T_0\,S/2AG}\text{, and} \tag{27}$$

$$\bar{T} = \frac{T}{T_0} = \frac{T}{4A^2 tG\,\theta_0/S} \tag{28}$$

The above relationship may be expressed in non-dimensionalized form as,

$$\bar{T} = 1+\frac{\alpha}{G}\left(\bar{\theta}-1\right)$$

$$\bar{\theta}_R = \left(\bar{T}-1\right)\left[\frac{G}{\alpha}-1\right]$$

or

$$\bar{\theta}_R = \left(\bar{\theta}-1\right)\left[1-\frac{G}{\alpha}\right] \tag{29}$$

As obtained in reference [30].

Experiments

The shear stress-strain curve for a mild steel specimen was plotted from a tensile stress-strain curve by using the relationships obtained in Appendix A. Thin tubes of square (25 mm × 25 mm) section having length and thickness of 200 mm and 2 mm respectively were made and subjected to torsion test on Avery Torsion Testing machine, in which the angle of twist is measured with a Vernier scale, giving an accuracy upto 0.1^0. The angles of twist were noted in the elasto-plastic regime for different test pieces. Experimental points are shown along with the theoretical curves in Figure 7. Details of the experiments are given in Appendix B.

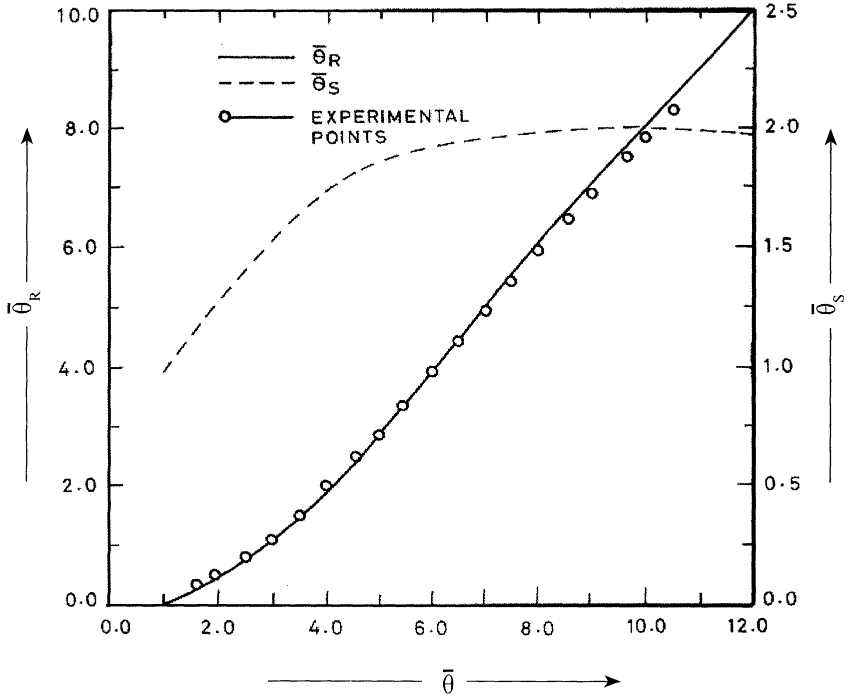

Figure 7: Non-dimensionalized Angle of Twist ($\bar{\theta}$) Versus Non-dimensionalized Residual Angle of Twist ($\bar{\theta}_R$) and Non-dimensionalized Springback Angle of Twist ($\bar{\theta}_S$)

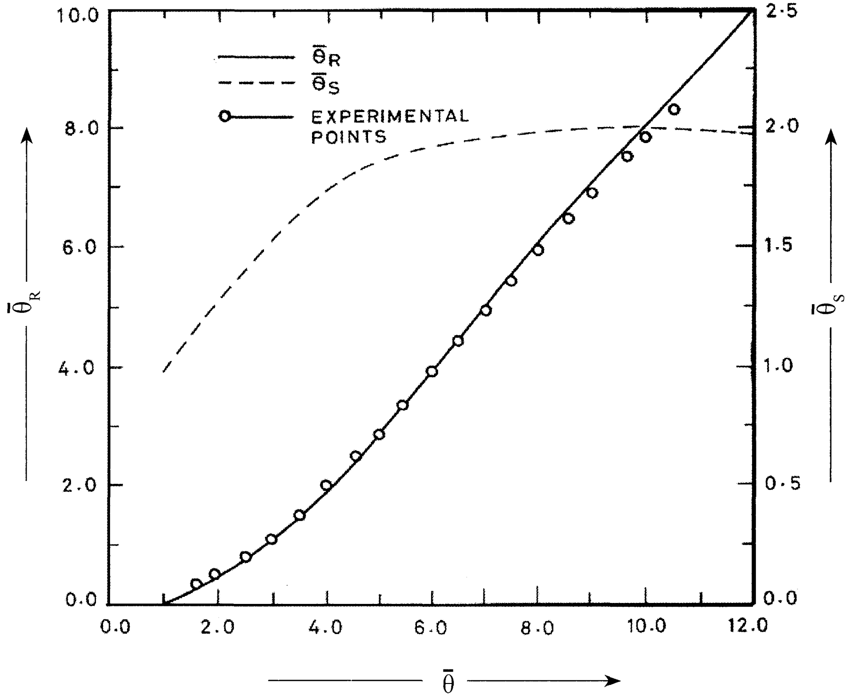

Figure 7: Non-dimensionalized Angle of Twist ($\bar{\theta}$) Versus Non-dimensionalized Residual Angle of Twist ($\bar{\theta}_R$) and Non-dimensionalized Springback Angle of Twist ($\bar{\theta}_S$)

Results and Discussion

It has already been shown in the analysis that when stress-strain relationship is idealized as bi-linear one, the theoretical expressions obtained from the general expression are exactly the same as those obtained in reference [36]. It is also seen from Figure 7 that the experimental results obtained are in close agreement with the results obtained by method obtained using the actual stress-strain curves. The differences in the theoretical and experimental values are due to the assumption made in the analysis that the relationship between displacement and strains for small deformation are also valid for the strain range under consideration. However, inspite of the differences the close agreement between the values obtained theoretically from the stress-strain curve and those obtained experimentally confirms the validity of the theoretical analysis. The method has the advantage that no approximation or idealization of the stress-strain curve is needed to get the results theoretically.

Conclusions

Based on the results presented above the following conclusions have been drawn:

1. The method provides values of springback angle of twist/residual angle of twist to satisfactory level of accuracy and is in close agreement with experimental results.
2. No approximation/idealization of the stress-stain curve is needed.
3. No torque versus angle of twist curve is drawn for obtaining the values of springback angle of twist.
4. The values of springback angle of twist can be obtained with same case whether the loading is kinematic or kinetic.

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APPENDIX-A

Though it is not easy to get the shear stress-strain relationship experimentally, it may be derived from the stress-strain curve from tensile test and the value of G as follows [5].

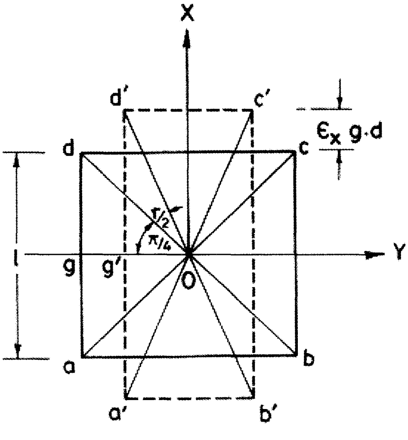

Figure 8: Deformation of Square Element abcd Due to Tension in the x-direction

Let a square element shown in Figure A1 be deformed to due to tension in x-direction. Let the strain in x-direction be ϵ_x, then the maximum shear strain will be at an angle $\pi/4$ with the X-axix, i. e. along the diagonal of the square. The angle is definitely equal to $[(\pi/4) + (\gamma/2)]$. So,

$$\tan\left[\frac{\pi}{4}+\frac{\gamma}{2}\right] = \frac{g'd'}{0\,g'} = \frac{g\,d\,(1+\epsilon_x)}{0\,g\,(1+\epsilon_y)} = \frac{(1+\epsilon_x)}{(1+\epsilon_y)} \tag{A1}$$

$$\epsilon_y = -\mu\,\epsilon_x$$

Taking $\mu = 0.5$ in plastic state of stress from equation (A1)

$$\frac{1+\tan\gamma/2}{1-\tan\gamma/2} = \frac{1+\epsilon_x}{1-0.5\epsilon_x}$$

or

$$\frac{1+\gamma/2}{1-\gamma/2} \approx \frac{1+\epsilon_x}{1-0.5\epsilon_x}$$

or

$$\gamma \approx 1.5\epsilon_x$$

so

$$\frac{d\gamma}{d\epsilon} = 1.5 \tag{A2}$$

$$\frac{d\sigma_x}{d\tau} = 2, \tag{A3}$$

Where τ is the maximum shear stress and it acts along, so from equations (A2) and (A3)

$$\frac{d\tau}{d\gamma} = \frac{1}{3}\frac{d\sigma_x}{d\epsilon_x} \tag{A4}$$

So, the shear stress-strain curve in the elasto plastic regime can be obtained from tensile stress-strain curve by using the above relationship.

Table A: Mechanical Properties of Tube Materials

Modulus of elasticity (E) (N / mm^2)	Modulus of rigidity (G) (N / mm^2)	Yield shear stress (τ_0) (N / mm^2)	Plastic modulus of rigidity (α) (N / mm^2)	α/G
2.1×10^5	8.24×10^4	108	6.1×10^3	0.074

APENDIX-B

Experimental Procedure

The present work is concerned with experimentally quantifying the torque, springback and residual angle of twist per unit length of thin mild steel tubes of square (25 mm × 25 mm) section having length and thickness of 200 mm and 2 mm respectively. Figure A shows the pictorial view of "Avery Torsion Testing Machine". The strain was applied to the specimen by worm and spur gearing was so arranged that the full load might be applied by hand without undue effort. The load was transmitted from the weighing spindle by means of a horizontal torque arm, mounted on antifriction bearings, which transmitted a vertical pull to the indicating unit. The load indicator was of self indicating cam-resistant type and the die carried two sets of graduations with pointer and chart edge to edge to avoid parallax. Capacity could be controlled by means of hand lever and a maximum load pointer mounted on a separate spindle provided a record of breaking loads. For the purpose of testing and gripping of tubes, a holder had been designed and fabricated which could be fitted in four-jaw type self gripping chuck and attached with the face plate holder.

With the help of hand wheel, a small torque was applied to the specimen that was noted directly from the indicator and corresponding angle of twist was also noted with the help of Vernier and protractor fitted in the machine. The torque was gradually increased up to elasto-plastic regions and corresponding

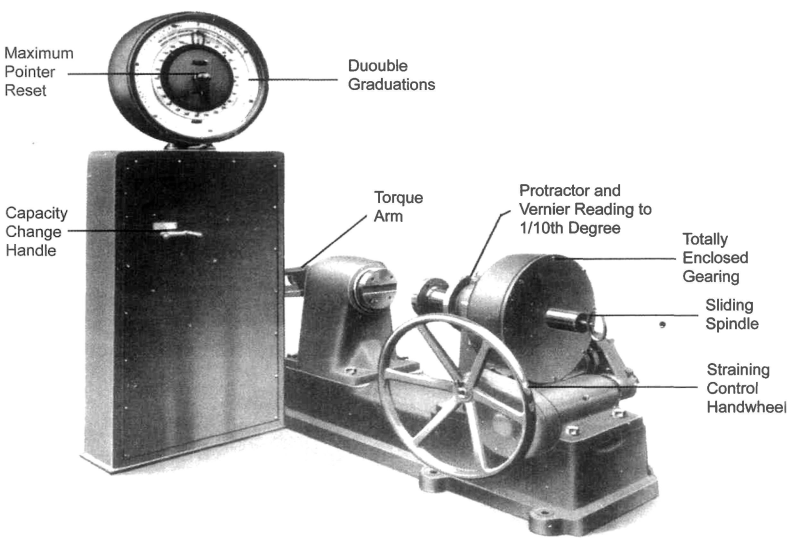

Figure A: Avery Torsion Testing Machine

angle of twists were noted. Now the torque was released slowly to zero and the residual angles of twist (θ_R) of the deformed tubes were noted. These deflection readings were again checked with the help of combination set just after setting the torque to zero. The procedure was repeated for number of specimens. Springback percentage and residual angle to twist per unit length in percentage were calculated for each specimen with the help of equations (24) and (25).